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Baseline model for energy system as two-sided market

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Preface

This report was done as a part of the Finnish national research project "Flexible Energy Systems" FLEXe, and it was funded by Tekes, the Finnish Funding Agency for Technology and Innovation, and the project partners. The report was written in the Finnish Environment Institute (SYKE).

Name of the report: Baseline model for energy system as two-sided market

Key words: two-sided market, platform, pricing, electricity

Summary

This report presents the idea of a two-sided platform market for flexibility in the electricity system. The changes happening today, larger share of variable renewable energy sources and larger penetration rate of smart metering in households, are possible drivers towards platform market, which combines industry actors in need of balancing services and electricity users able to provide balancing.

Based on the economic theory, the report goes through the main insights regarding the optimal pricing structure in the two-sided markets. Characteristic for the two-sided market is the cross network effect: the utility of group A members depends on the quantity of group B agents in the platform and the utility of group B members depends on the quantity of group A agents in the platform. Given the cross network effects, setting the price structure such that both sides get on board is highly important for the platform operator. For the market regulator, understanding the dynamics of two-sided markets is essential, as price below marginal costs may not necessarily signal predation and price above marginal costs may not necessarily signal market power utilization.

An illustrative example of pricing in the two-sided market is presented after the theory presentation. The aim of the example is to underline the components, which the platform operator has to take into account in the pricing decision: own and cross side elasticities, group sizes and price set for the other side of the platform.

Helsinki, June 2016

Contents

1 Introduction

"In general, the innovations and modern customer services in electricity retailing have not kept pace with peer industries, such as telecom and banking."

Fortum Energy Review, March 2015

Traditionally, economies of scale have dominated the structuring of power systems. Utilities have been able to increase efficiency by building bigger units. Transmission and distribution operators have done their job by providing enough grid capacity and taking care of the grid balance. Consumers have been rather passive components at the other end of the vertically connected electricity system. This system structure has provided consumers a secured supply of moderately priced electricity. In addition, electricity has been a good which households have been able to consume without thinking about its origin or the technological challenges related to the maintenance of the security of supply.

This situation is about to change due to two main drivers: smart devices and emission reduction targets. The advancements in technology make it possible to measure and exchange electricity and consumer data in real time. This implies that consumers can be handled as an active components in the system and the ability to serve heterogeneous customers has improved. On the other hand, the clear need to reduce carbon dioxide emissions globally means that investments have to be guided into zero-emission renewable power technologies. The fact that renewables like wind and solar power cannot be dispatched like thermal plants and there are uncertainties related to their production profiles implies that more flexibility providing components will be needed in the electricity systems. However, renewables have depressed electricity prices and the incentives for investing into traditional flexible thermal technologies have decreased. Consequently, the need for new ways to offer flexibility has increased. The obvious direction is to look at electricity consumers.

The sharing economy is a hot topic in many industries currently. For example, completely new operators providing intermediation services through platforms have arisen in taxi services (Uber) and in accommodation (Airbnb). These operators create value by matching people with underutilized assets and people in need of the services these assets can provide. Potentially, new platform operators can increase the social welfare by increasing the utilization rate of existing assets. In addition, the long-run effect can change investment decisions, as higher fixed cost assets investments can be reasoned through higher hours in use (a high quality assets can be afforded given the increased utilization rate).

Essentially, the advancement of technology exposes incumbent firms into the threat of new market entrants. If there exists underutilized assets, new operators are able to hop into the industry and provide an application which acts as an intermediary between the group in need of the asset and the group able to provide the asset. Operator makes profit by taking a share of the added value.

Arguably, there exists potential areas in the electricity markets for a greater asset utilization. For example, consumers have had a rather passive role in the system, but new demand response schemes, automatically adjusted loads and illustrative user interfaces and applications open up a more active role for consumers. This kind of development means that utilization of existing flexibility potential increases and incentives for future investments (e.g. batteries) change as consumers and households are given a change to be remunerated based on the flexibility services they are able to provide.

In this report we present the idea of flexibility market platform, which intermediates between producers in need of balancing services and households able to provide flexibility. In the spirit of Weiller and Pollit (2013) we present the idea of this intermediation as a platform two-sided market. Based on the economic theory of multi-sided markets, this report studies the consequences for pricing in two-sided market models. We present illustrative theoretical examples of choosing the optimal price structure in the two-sided market setting[1].

The main message of this report is that in a two-sided market setting, using the traditional one-sided logic with respect to pricing and regulation will lead to inefficient outcomes. Figure 1 illustrates the cross network effect on both sided of the two-sided platform, i.e. the decision of group A agents to join the platform depends on the quantity of group B agents who have joined the platform, and vice versa. Thus, platform operator has to choose the price structure such that *both sides get on board*.

[1] For a more detailed study of aggrerator business in power systems see e.g. Ikäheimo et. al. (2010).

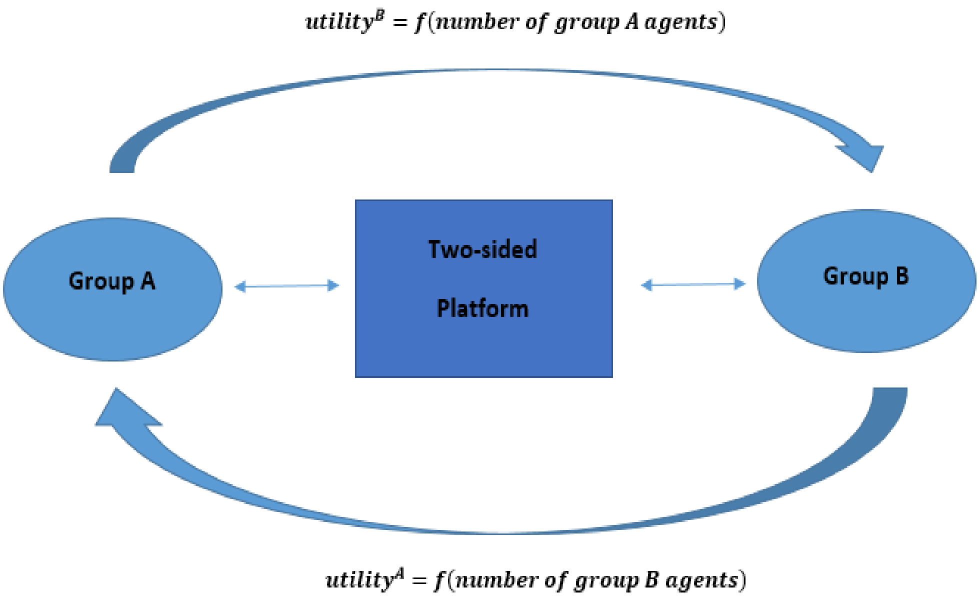

Figure 1 Cross-network effects of two-sided market.

Even for the seemingly simple market setting in Figure 1, setting the right price structure is not an uncomplicated task. As is noted e.g. Argentesi and Filistrucchi (2007), the platform operator has to take into account four different price elasticities[2]: the elasticity of group *A* participation with respect to group *A* price, the elasticity of group *A* participation with respect to group *B* price, the elasticity of group *B* participation with respect to group *A* price and the elasticity of group *B* participation with respect to group *B* price. Essentially, the interaction between the two sides has to be taken into account in two-sided market pricing study.

2 Drivers for the change in the electricity markets

The main drivers pushing the electricity markets into a structural shift are increasing share renewable energy sources and a greater role of information and communication technologies. Renewable energy sources like wind and solar produce electricity intermittently with low marginal costs. Consequently, a larger penetration of these technologies implies that the future market model will move from marginal cost pricing into pricing models which compensate for producer and user flexibility. Technology makes the transition possible by allowing automatic load balancing and a real-time measurement of electricity flows.

[2] Price elasticity refers to the percentage change in quantity with respect to the percentage change in price.

Climate change is a global problem and solving it requires globally coordinated answers. In December 2015, in the Paris climate conference governments agreed to limit the global warming to below 2°C above the pre-industrial level. This deal implies a huge increase in the investments for zero-emission renewable energy sources. However, with intermittently producing renewable energy sources the challenges with regards to grid balance and security of supply increase. As the supply of wind and solar power is variable and uncertain, the integration of these technologies causes integration costs at the system level.

Hirth et. al. (2015) show that integration costs of variable renewables are non-neglible and should be taken into account in the welfare-optimal generation mix studies. Additionally, Weber (2010) shows that with increasing wind power capacities in the system, the required balancing capacity gets dominated by the wind power estimation error. The adjustment costs C_{adj} can be written as a function of the relative wind forecast error $E_{wind,rel}$, the installed wind power capacity Cap_{wind} and the slope m of the short-term balancing price function:

$$C_{adj} = m * (E_{wind,rel} * Cap_{wind})^2 \quad (1)$$

The adjustment costs increase quadratically with installed wind power capacity. Thus, decreasing the price slope of short-term balancing market would decrease the adjustment costs considerably. Weber (2010) suggest different possibilities for improving the functioning of intraday market, e.g. change from day-ahead spot auction to continuous spot trading or obliging market partners to bid in the intraday market. An additional possibility, not raised in Weber (2010), would be to get consumers more involved in the short-term balancing market. With a two-sided flexibility market, by getting both producers and consumers on board, balancing market liquidity could be improved and so the slope m of balancing price function could be reduced.

The advancements in ICT and the introduction of smart grids are the enablers in the transition towards market models with active consumer participation. According to European Commision report (European Commission, 2014a) about 195 million smart meters will be installed by 2020 (representing 72% of EU-27 consumers). In the Nordics, the expected diffusion rate of smart meters for Denmark, Finland and Sweden is 100 percent by 2020. In Norway, at least 80 percent of metering point has to meet the functional automatic metering requirements by the beginning of 2016 (NordREG, 2014). All in all, the implementation of smart metering infrastructure is a widely discussed issue worldwide (Leiva et. al., 2016).

Given the aforementioned drivers for change in the electricity system, transition into low-carbon system and the increasing share of smart-meter households, Weiller and Pollit (2013) present two possible platform mediated cases: a balancing service management and an electric vehicle charge management. These cases are based on the UK market contexts. The position of the platform entrants in shown in Figure 2.

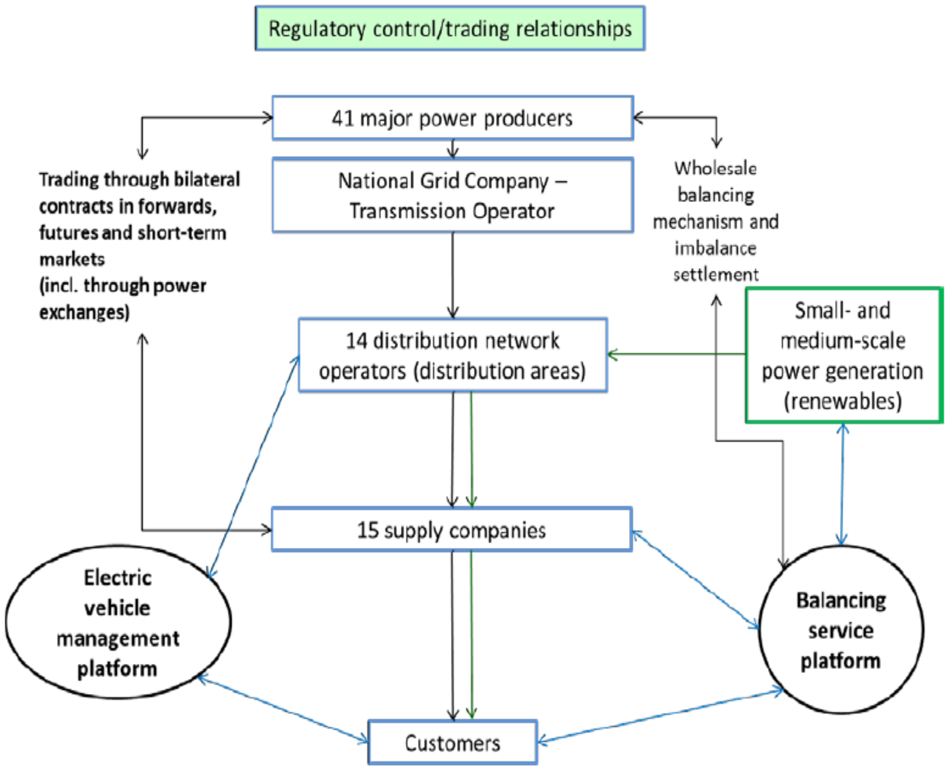

Figure 2 Two platform entrants: balancing services and electric vehicle management (cases based on UK market context). (source: Weiller and Pollit, 2013)

The balancing services example is based on the platform provider managing the electricity load from households and commercial consumers. This service is sold to electricity market actors who benefit from the balancing, e.g. generators, energy service companies and distribution network operators. Thus, the platform can price the service for both sides of the market. The platform provider can be for example an energy service company, a data management company or a company from retail or finance industry. However, a possible barrier for entry is the availability of customer data.

The electric vehicle aggregator example presents an idea of operator managing the charge/discharge cycle of electric vehicles optimally for the electricity system. As Figure 2 shows, the platform operator acts as a mediator between the electric vehicle drivers and power supply companies, power producers and DNOs. This charging/discharging service helps electric vehicle owners to capture the whole value of the batteries, and thus improve the incentive of becoming an electric vehicle owner.

The need for the balancing services can be expected to get stronger in the future, as suppliers with unforecastable generation will benefit from a large participation rate of consumers in demand response pools. Consequently, Weiller and Pollit (2013) conclude that the transition towards low-carbon electricity system makes platform business models and two-sided pricing strategies a topical subject in electricity markets.

3 Two-sided platform market literature

Two-sided platform operator makes profit by providing a real or a virtual marketplace for two distinct groups who need each other (externality), but the transaction does not take place directly because of the high transaction costs. As Evans (2011) notes, platform solves the externality by providing a transaction costs minimizing technology.

A general description for two-sided market is provided in Rysman (2009):

1) two groups of agents interacting through a platform
2) the decision of each group of agents affects the outcomes of the other group of agents.

As an example, video game systems require two sets of agents, consumers and video game developers, and the platform is the console producer. Playstation or Xbox will not be interesting for the consumers without quality games, and vice versa, game developers are not going to use resources for developing games without a large mass of potential consumers on the other side of the platform. Another example is payment card system. Visa or Master Card need both a large group of consumers and a wide net of merchants accepting their cards.

In the economics literature the description is more detailed. Weyl (2010) lists three features necessary for choosing the two-sided market modeling framework:

i. Multi-product firm: the platform provides different services/products to the two sides. Additionally, it is able to charge different prices.
ii. Cross network effects: utility of agent on side A depends on the participation on side B.

iii. Bilateral market power: platform are price setters on both sides, i.e. monopolistic or oligopolistic.

These conditions imply that the market interactions are possibly more complicated than in the traditional one-sided setting. These interactions have a huge effect on the pricing decision that platforms operating in two-sided markets face.

In the electricity markets this kind of setting arises if there are few operators providing intermediation services using a common platform through which two groups (e.g. producers and households) can interact. The prices have to be set differently for both sides such that both sides get on board.

Next we highlight the main issues regarding to price structure with monopoly platform examples. Additionally, other strategy decisions as well as regulation policy guidelines are presented briefly.

3.1 Monopoly platform pricing

This section presents the monopoly platform pricing rule derived in the two essential two-sided markets papers: Rochet and Tirole (2003) and Armstrong (2006). Here, the two sides are *B* (buyers) and *S* (sellers). A platform member on side $I = \{B, S\}$ derives a membership benefit α^I and a per-transaction benefit β^I from the platform. The general assumption is that the utility from joining the platform for an agent *i* on side $I = \{B, S\}$ is

$$u_i^I = \alpha_i^I + (\beta_i^I - p^I)n^J - P^I(n^J) \tag{2}$$

where n^J is the fraction of members on the other side who decide to participate, p^I is a transaction price (charged by the platform) and P^I is the entrance fee to the platform.

3.1.1 Rochet and Tirole (2003): Platform competition in two-sided markets

Rochet and Tirole (2003) model the platform market with no membership benefits ($\alpha_i^I = 0$) and no membership costs ($P^I = 0$). Here, the utility for the agent on side I is

$$u_i^I = (\beta_i^I - p^I)n^J \tag{3}$$

Importantly, u_i^I depends on the number of agents on the other side n^J (cross network effect), but the decision to join the platform is independent of n^J. Thus, Rochet and Tirole (2003) use the “quasi-demand functions”

$$N^B = \Pr(\beta^B \geq p^B) = D^B(p^B),\ N^S = \Pr(\beta^S \geq p^S) = D^S(p^S) \tag{4}$$

for buyers and for sellers, respectively. The authors assume that each pair $D^S(p^S)\,D^B(p^B)$ leads to one transaction. Now, the monopoly platform’s profit function is

$$\pi = (p^B + p^S - c)\, D^S(p^S)\, D^B(p^B) \tag{5}$$

Setting prices such that profit are maximized leads to the following condition

$$\frac{dD^B(p^B)}{dp^B}/D^B(p^B) = \frac{dD^S(p^S)}{dp^S}/D^S(p^S) \tag{6}$$

Plainly, *the proportional changes in quasi-demands with respect to price are equated at the optimum*. By introducing the following elasticities of quasi-demands,

$$\varepsilon^B = -\frac{p^B}{D^B}\frac{dD^B(p^B)}{dp^B},\ \varepsilon^S = -\frac{p^S}{D^S}\frac{dD^S(p^S)}{dp^S} \text{ and } \varepsilon = \varepsilon^B + \varepsilon^S \tag{7}$$

price allocation between sides B and S can be written as

$$p^B = \frac{\varepsilon^B}{\varepsilon - 1}c \text{ and } p^S = \frac{\varepsilon^S}{\varepsilon - 1}c \tag{8}$$

Consequently, *the mark-up over marginal cost for sides B and S depend on the own elasticity of quasi-demand and on the total elasticity of quasi-demand* (which is assumed to be larger than 1).

3.1.2 Armstrong (2006): Competition in two-sided markets

Armstrong (2006) models the platform market with the interaction term depending only on the side of which the agent is on ($\beta_i^I = \beta^I$). In addition, only the cost of joining the platform is taken into account ($p^I = 0$), i.e. the platform charges a lump-sum fee P^I. Initially also the fixed benefit α_i^I is left out of the study. Now, the utilities for the agents on sides B and S are

$$u^B = \beta^B n^S - P^B \text{ and } u^S = \beta^S n^B - P^S \tag{9}$$

Term β^I describes the benefit that the agent in group I gets from the interaction with each agent from the other group (cross network effect) and platform price P^I affects agent's utility as well. Armstrong (2006) introduces increasing functions $\theta_B(.)$ and $\theta_S(.)$ which specify the number of agents participating in the platform as a function of utility

$$n^B = \theta_B(u^B) \text{ and } n^S = \theta_S(u^S) \tag{10}$$

The operator has per-agent costs c^B and c^S from attaching and serving agents from groups B and S into the platform. Therefore, the platform profit function is

$$\pi = n^B(P^B - c^B) + n^S(P^S - c^S) \tag{11}$$

With respect to utilities, the profit function can be written as

$$\pi(u^B, u^S) = \theta_B(u^B)[\beta^B\theta_S(u^S) - u^B - c^B] + \theta_S(u^S)[\beta^S\theta_B(u^B) - u^S - c^S] \tag{12}$$

where the prices for groups B and S are written implicitly as $P^B = \beta^B\theta_S(u^S) - u^B$ and $P^S = \beta^S\theta_B(u^B) - u^S$, respectively. The price structure maximizing platform profit is

$$P^B = c^B - \beta^S n^S + \frac{\theta_B(u^B)}{\theta_B'(u^B)} \text{ and } P^S = c^S - \beta^B n^B + \frac{\theta_S(u^S)}{\theta_S'(u^S)} \tag{13}$$

It follows that, *the optimal price for agents in group I equals the cost of servicing c^I, less the cross network benefit to other group's agents, plus the sensitivity of group's participation.*

Using the following price elasticities of demand for a given level of participation by the other group

$$\epsilon^B(P^B|n^S) = \frac{P^B\theta_B'(\beta^B n^S - P^B)}{\theta_B(\beta^B n^S - P^B)} \text{ and } \epsilon^S(P^S|n^B) = \frac{P^S\theta_S'(\beta^S n^B - P^S)}{\theta_S(\beta^S n^B - P^S)} \tag{14}$$

the optimal price structure can be written as follows

$$\frac{p^B-(c^B-\beta^S n^S)}{p^B}=\frac{1}{\epsilon^B(p^B|n^S)} \text{ and } \frac{p^S-(c^S-\beta^B n^B)}{p^S}=\frac{1}{\epsilon^S(p^S|n^B)}. \quad (15)$$

Comparing the above price structure rule to the traditional Lerner elasticity pricing rule

$$\frac{p-mc}{p}=\frac{1}{\varepsilon}, \quad (16)$$

where p is price, mc is marginal cost and ε denotes price elasticity, one sees that Armstrong (2006) profit maximizing price structure takes into account also the cross network effect.

All in all, *the monopoly platform pricing can lead to group I being subsided by the platform, i.e. a case where $p^I < c^I$*. This situation can take place if the external benefit group I agent brings to the other group's agents is large enough and the elasticity of demand of group I agents is high.

3.2 Competing platforms

Literature of the economics of two-sided market includes also multiple platform models. When agents use several platforms they are said to “multi-home” and when agents use only one platform are said to “single-home”. When one group single-homes and other group multi-homes the asymmetric price structure between the groups may be amplified compared to the monopoly platform. In this “competitive bottleneck” configuration the platform with single-homing agents is in a monopoly position over the access for these customers. Hence, higher prices can be charged from the multi-homing agents, whereas single-homing agents are the ones to be attracted through lower prices. All in all, the profits collected from multi-homing side flow to a large extend to the single-homing side (Armstrong, 2006).

In some markets it might be that not all of the agents are informed about all of the prices. One side may simply be uninformed about the prices set to other side. In this case, the expectations on the uniformed side do not respond to platform price changes. As Hagiu and Halaburda (2014) show, different levels of information lead to different market outcomes. If the platforms operating in the market have market power they will prefer as informed users as possible. This way the users' response to price changes and platform can steer the outcome into profit maximizing equilibrium. On the other hand, competing platforms would achieve higher profits with uninformed users. More information tends to intensify price competition and platform profits will be sacrificed in this setting.

Besides setting the right price structure, other strategic decisions for firms competing in two-sided market environment are how to cope with the returns to scale related to network externalities and how to avoid attacks from platforms with overlapping groups. Eisenmann, et. al. (2006) name the first challenge as “winner-take-all dynamics” and the second challenge as “thread of envelopment”.

Winner-take-all dynamics arise from the network externalities. If platform is able to collect a certain amount of customers on side one, it seems an attractive choice for the other side which appreciates the interaction possibilities. Furthermore, attracting the other side customers makes the platform even more attractive for agents on the side one. Thus, there exists a tendency for two-sided market to be served only by one platform. A single platform structure rises most probably if the network effects are strong and if the cost of multi-homing for at least one side is high.

The thread of envelopment refers to a situation where a platform with overlapping user base enters a new market by using the shared customers. Here, a platform operating in the market is “enveloped” by the platform from an adjacent market. A multiplatform operator might be able to offer a better functionality by combining the benefits the user enjoys from using multiple platforms from different markets. Networked markets are well suited for envelopment, and the blurring of market lines leads markets to converge around certain platforms. As an example, Eisenmann, et. al. (2006) mention mobile phones which include the functionalities of music and video players, PCs and credit cards.

3.3 Regulation

Using a one-sided logic in two-sided markets will lead to wrong conclusions by market regulators. In two-sided markets there are two distinguishable decisions to be made for the platform operator: the price structure (price for group A and price for group B agents) and the price level (sum of two prices). Hence, the traditional “price equals marginal costs” – rule describing competitive markets cannot be applied, as both price level and structure has to be included in the regulator’s ruling decision.

Given this insight, Wright (2004) lists eight fallacies which can arise if one uses the one-sided logic in two-sided markets. The most important guideline is that an efficient price structure does not imply that prices should necessarily reflect the marginal costs. The network externalities in two-sided markets imply that the surplus one side enjoys from getting an additional agent on the other side on board should be reflected also in the prices. Depending on the cross-side externalities, the efficient price structure may consist of prices being below or above marginal costs. Consequently, a high price-cost margin on one side does not necessarily imply market power and a price below marginal cost does not necessarily signal predatory pricing.

Behringer and Filistrucchi (2015) use this pricing logic and extend the classical predatory pricing metric, so called Areeda-Turner rule, into two-sided markets. According to Areeda and Turner (1975), a price set below marginal costs reads as a sign of predation. If marginal cost is hard to estimate, the authors suggested that price below average variable costs can be used as the predatory pricing metric as well. In the two-sided markets the same logic still applies: if the platform monopoly is making an overall loss at the margin, then predatory pricing should be suspected. However, for two-sided markets, price can be below marginal costs for the agents on the other side, if this price structure reflects the positive cross-side externality these agents impose for the other side. Consequently, the platform makes up the loss by pricing the other side above marginal costs.

A two-sided Areeda-Turner rule takes into account the profit margin for each side, and, importantly, adds the cross-side externality effect into the equation. Thus, for a two-sided market with groups A and B, demands Q^A and Q^B and cost function $C(.)$, prices for side A signal predation, if

$$\left(p^A - \frac{\partial C}{\partial \hat{Q}^A}\right) + \left(p^B - \frac{\partial C}{\partial \hat{Q}^B}\right)\frac{\partial Q^A}{\partial Q^B} < 0 \tag{18}$$

and prices for side B signal predation, if

$$\left(p^B - \frac{\partial C}{\partial \hat{Q}^B}\right) + \left(p^A - \frac{\partial C}{\partial \hat{Q}^A}\right)\frac{\partial Q^B}{\partial Q^A} < 0 \tag{19}$$

It follows that, *prices should be suspected to be predatory, if the sum of two prices weighted by the marginal cross-network effect is lower than the sum of marginal costs weighted by the marginal cross-network effect.*

Finally, in line with the original Areeda-Turner (1975) suggestion, Behringer and Filistrucchi (2015) propose that if the marginal costs are hard to estimate, a more simple rule based on average variable cost (AVC) states that, prices for side A signal predation, if

$$(p^A - AVC^A) + \frac{Q^B}{Q^A}(p^B - AVC^B) < 0 \qquad (20)$$

and prices for side B signal predation, if

$$(p^B - AVC^B) + \frac{Q^A}{Q^B}(p^A - AVC^A) < 0 \qquad (21)$$

Again, the rule takes into account the sum of prices and the sum of average variable costs weighted by the cross-network effect.

4 Example on two-sided market model

In this section a two-sided market example is presented. The example is based a logit type of two-sided model. The example consist of two groups:

- Industry (group *A*): given the imbalance with regards to planned and realized production (e.g. wind and solar produce power intermittently) and forecasted and actual demand, group *A* members operating in the electricity system need balancing power services.
- Provider (group *B*): small scale (e.g. households) and large scale consumers (e.g. industry) can provide balancing services by transferring consumption based on the balancing needs.

A monopoly operator acts as an intermediary between the two groups by providing a platform through which the two sides can interact. The costs for the operator from adding and servicing an additional group A or group B agent are c^A and c^B, respectively. Operator charges a platform joining fee p^A for group A and p^B for group B. After agents from both groups have joined the platform, they are able to interact without having to pay any additional costs for the platform, i.e. the platform operator sets only the membership fee. The two-sided market model is presented in Figure 3.

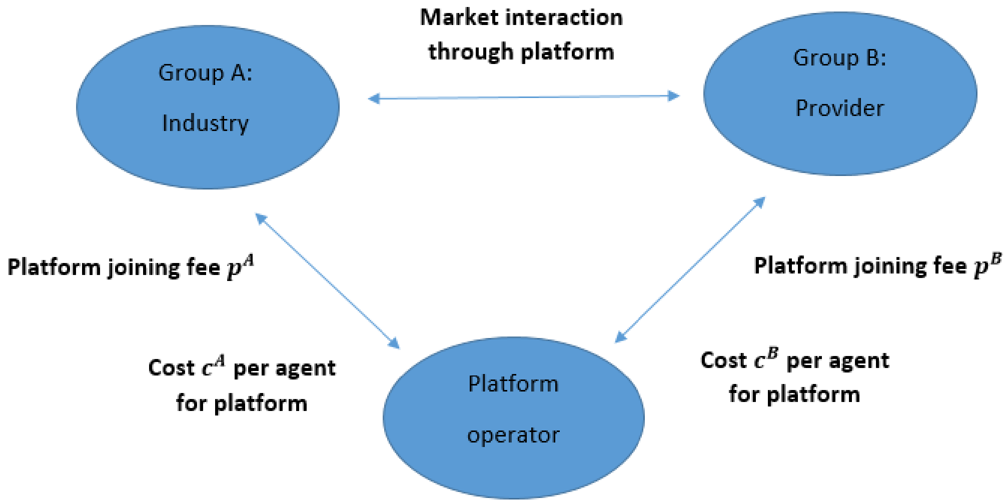

Figure 3 Platform in flexibility market.

The utility for an agent i from the industry (group A) side from joining the platform is written as

$$u_i^A = \alpha^A p^A + \beta^A q^B + \varepsilon_i^A \tag{22}$$

The first part $\alpha^A p^A$ shows how the platform price p^A affects agent's utility. Parameter α^A measures the disutility of platform price. The second part $\beta^A q^B$ marks the utility of having q^B agents from the group B in the platform. Parameter β^A measures the utility of interacting with the agents of the other group (cross-side externality). Household taste shock ε_i^A is assumed to be an i.i.d. type I extreme value.

Similarly, the utility for an agent i from flexibility provider (group B) side from joining the platform is written as

$$u_{ij}^B = \alpha^B p^B + \beta^B q^A + \varepsilon_i^B \tag{23}$$

Again, for agents in the producer side, parameter α^B measures the disutility of platform price and parameter β^B measures the utility of interacting with the agents from group A and taste shock ε_i^B is assumed to be an i.i.d. type I extreme value.

Group A and group B agents are able to choose the outside option, where they stay out of the flexibility platform. In this case the utility is zero plus the taste shock. Given these assumptions, the market share functions for platform j can be written in the logit form[3]:

$$S^A = \frac{\exp(\alpha^A p^A + \beta^A q^B)}{1+\exp(\alpha^A p^A + \beta^A q^B)} \tag{24}$$

$$S^B = \frac{\exp(\alpha^B p^B + \beta^B q^A)}{1+\exp(\alpha^B p^B + \beta^B q^A)} \tag{25}$$

It should be emphasized that q^A, the participation of group A agents in the platform, affects the share of group B agents joining the platform, and vice versa. As a result, setting total group populations as M^A and M^B, platform group sizes can be written as

$$q^A(p^A, q^B) = S^A M^A \tag{26}$$

and

$$q^B(p^B, q^A) = S^B M^B \tag{27}$$

Now, monopoly platform's profit maximization problem can be written as

$$\max_{p^A, p^B} \pi = (p^A - c^A)\, q^A(p^A, q^B) + (p^B - c^B)\, q^B(p^B, q^A) \tag{28}$$

The first order conditions are

$$\frac{\partial \pi}{\partial p^A} = q^A + (p^A - c^A)\frac{\partial q^A}{\partial p^A} + (p^B - c^B)\frac{\partial q^B}{\partial p^A} = 0 \tag{29}$$

for platform price p^A and

$$\frac{\partial \pi}{\partial p^B} = (p^A - c^A)\frac{\partial q^A}{\partial p^B} + q^B + (p^B - c^B)\frac{\partial q^B}{\partial p^B} = 0 \tag{30}$$

for platform price p^B.

[3] For a detailed description of discrete choice modeling, see Train (2009).

The mark-up rules can be derived from the first order conditions as follows,

$$(p^A - c^A) = -p^A \left(\frac{\partial q^A}{\partial p^A}\frac{p^A}{q^A}\right)^{-1} - (p^B - c^B)\left(\frac{\partial q^B}{\partial p^A}\frac{p^A}{q^B}\right)\left(\frac{\partial q^A}{\partial p^A}\frac{p^A}{q^A}\right)^{-1}\frac{q^B}{q^A} \tag{31}$$

and

$$(p^B - c^B) = -p^B \left(\frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B}\right)^{-1} - (p^A - c^A)\left(\frac{\partial q^A}{\partial p^B}\frac{p^B}{q^A}\right)\left(\frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B}\right)^{-1}\frac{q^A}{q^B}. \tag{32}$$

The own-side price elasticities can be presented as

$$\varepsilon_{own}^A = \frac{\%\ change\ in\ q^A}{\%\ change\ in\ p^A} = \frac{\partial q^A}{\partial p^A}\frac{p^A}{q^A} \tag{33}$$

and

$$\varepsilon_{own}^B = \frac{\%\ change\ in\ q^B}{\%\ change\ in\ p^B} = \frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B} \tag{34}$$

Additionally, the cross-side price elasticities can be presented as

$$\varepsilon_{cross}^A = \frac{\%\ change\ in\ q^A}{\%\ change\ in\ p^B} = \frac{\partial q^A}{\partial p^B}\frac{p^B}{q^A} \tag{35}$$

and

$$\varepsilon_{cross}^B = \frac{\%\ change\ in\ q^B}{\%\ change\ in\ p^A} = \frac{\partial q^B}{\partial p^A}\frac{p^A}{q^B} \tag{36}$$

As a results, the monopoly platform mark-ups can be presented in the following form:

$$(p^A - c^A) = -p^A \frac{1}{\varepsilon_{own}^A} - (p^B - c^B)\varepsilon_{cross}^B \frac{1}{\varepsilon_{own}^A}\frac{q^B}{q^A} \tag{37}$$

and

$$(p^B - c^B) = -p^B \frac{1}{\varepsilon_{own}^B} - (p^A - c^A)\varepsilon_{cross}^A \frac{1}{\varepsilon_{own}^B}\frac{q^A}{q^B} \tag{38}$$

As can be seen from equations (37) and (38), the profit maximizing platform mark-up decision for one side is affected by

- the own-side price elasticity,
- the mark-up set for the other side,
- the cross-side price elasticity and
- the fraction of group sizes.

Note that the price elasticities here include the direct effect of price change and the indirect effect due to the cross-network effect. When platform increases the price p^A, the share group A agents who join the platform will decrease (direct effect). In addition, lower participation from group A makes the platform less beneficial for group B agents, and the share of group B agents joining the platform goes down. This in turn affects group A agents' decision to join, etc. (indirect effect).

All in all, given the cross-network effect, relatively simple market settings will lead into pricing rules which take into account the feedback effects of two-sided markets. Theoretically, given that the cross-network effects are not too large[4], the two-sided market model presented here can be solved by using the reduced form demands $\widetilde{q^A}(p^A, p^B)$ and $\widetilde{q^B}(p^A, p^B)$ and the implicit function theorem. The first order conditions are derived in Appendix.

5 Application: pricing in flexibility market

The logit two-sided market model presented in Section 4 is used as an example of platform pricing for flexibility market. This is an illustrative example and the model assumptions are chosen such that the main intuition with regards to optimal platform mark-up decision is clarified.

5.1 Two groups: Industry and flexibility providers

We assume equal group sizes[5] for the industry $M^A = 1000$ and for the flexibility providers $M^B = 1000$. The cost of adding an agent into the platform is

[4] Filistrucchi and Klein (2013) show that demands can be written as functions of both prices, if the product of indirect network effect is not too large: $\left|\frac{\partial q^A}{\partial q^B}\frac{\partial q^B}{\partial q^A}\right| < 1 \forall p^A, p^B$.

[5] This assumption is made in order to keep the example as illustrative as possible. A more reasonable assumption concerning balancing market would set group *B* (the flexibility providing household group) considerable larger than group *A* (industry group in need of flexibility services).

set equally for both sides: $c^A = c^B = 10$. Similarly, the price sensitivity parameters for groups A and B are equal: $\alpha^A = \alpha^B = -0.1$.

Network effect parameters are chosen such that groups differ with respect to their cross network effects. For industry agents (group A), the benefit of having an additional agent on the flexibility provider side (group B) is larger than the benefit for provider side (group *B*) agents arising from additional agent on the industry side (group *A*). Here, parameter values $\beta^A = 0.03$ and $\beta^B = 0.02$ are chosen.

Consequently, the utility functions are now

$$u_i^A = -0.1p^A + 0.003q^B + \varepsilon_i^A \quad (39)$$

for the industry side, and

$$u_i^B = -0.1p^B + 0.002q^A + \varepsilon_i^B \quad (40)$$

for the flexibility provider side.

Using the theory presented in chapter 4, the prices listed below in Table 1 are set for the two sides by the monopoly profit maximizing platform.

Table 1. Platform pricing.

	group A	**group B**
platform price	$p^A = 18.318$	$p^B = 15.745$
mark-up	$p^A - c^A = 8.318$	$p^B - c^B = 5.745$

In this example, platform makes positive profit on both sides. However, because of the stronger cross-network effect of group A, it is optimal for the platform to charge a higher price from group A agents. As Table 2 illustrates, the optimal pricing condition in this setting leads to symmetric participation on both sides.

Table 2. Platform market share.

	group A	group B
share [%]	$S^A = 0.257$	$S^B = 0.257$
Group size	$q^A = S^A M^A = 257$	$q^B = S^A M^A = 257$

Table 3 lists the own- and cross-side elasticities on the profit maximizing price point. Clearly, the difference in the cross-side externality parameters induces the platform to operate in the different sensitivity points with respect to prices for the two groups.

Table 3. Own- and cross-price elasticities.

	group A	group B
price, group A	$\varepsilon_{own}^A = \frac{\partial q^A}{\partial p^A}\frac{p^A}{q^A} = -1.742$	$\varepsilon_{cross}^B = \frac{\partial q^B}{\partial p^A}\frac{p^A}{q^B} = -0.859$
price, group B	$\varepsilon_{cross}^A = \frac{\partial q^A}{\partial p^B}\frac{p^B}{q^A} = -0.667$	$\varepsilon_{own}^B = \frac{\partial q^B}{\partial p^B}\frac{p^B}{q^B} = -1.498$

The platform can operate on the more price sensitive area with group *A* (ε_{own}^A =-1.742) than with group *B* ($\varepsilon_{own}^B = -1.498$). Additionally, although the cross-side elasticities are not as strong as the own-price elasticities, they have a clear impact on the pricing decision.

5.2 Three groups: Industry and two groups of flexibility providers

In this example, we divide the flexibility provider group *B* evenly into two sub-groups: *B1* consists of high flexibility electricity users (group size $M^{B1} = 500$) and *B2* consists of low flexibility electricity users (group size $M^{B2} = 500$). Here, industry group agents receive a larger utility from having an additional agent of the high flexibility *B1* sub-group in the platform (cross-side externality parameter 0.004) than from having an additional agent of the low flexibility *B2* sub-group in the platform (cross-side externality parameter 0.002).

The utility function for agent *i* in the industry side (group A) is

$$u_i^A = -0.1p^A + 0.004q^{B1} + 0.002q^{B2} + \varepsilon_i^A \tag{41}$$

For the two flexibility providing sub-groups the utility function for agent *i* in the sub-group *B1* is

$$u_i^{B1} = -0.1p^{B1} + 0.002q^A + \varepsilon_i^{B1} \tag{42}$$

and the utility function for agent *i* in the sub-group *B2* is

$$u_i^{B2} = -0.1p^{B2} + 0.002q^A + \varepsilon_i^{B2} \tag{43}$$

Now, the quantity of group A agents the operator manages to get to join the platform is a function of own price p^A and the quantity of flexibility providing agents q^{B1} and q^{B2}: $q^A(p^A, q^{B1}, q^{B2})$. The quantities of group B1 and B2 agents are functions of own prices and the quantity of group A agents: $q^{B1}(p^{B1}, q^A)$ and $q^{B2}(p^{B2}, q^A)$. However, the cross network effect implies that all of the prices are included in the reduced form quantity functions[6]: $\widetilde{q^A}(p^A, p^{B1}, p^{B2})$, $\widetilde{q^{B1}}(p^A, p^{B1}, p^{B2})$ and $\widetilde{q^{B2}}(p^A, p^{B1}, p^{B2})$.

The profit maximization first order conditions lead into the following mark-up rules:

[6] For group *A* the quantity is $q^A\left(p^A, q^{B1}(p^{B1}, q^A), q^{B2}(p^{B2}, q^A)\right)$. Thus, for groups *B1* and *B2* the quantities are $q^{B1}\left(p^{B1}, q^A\left(p^A, q^{B1}(p^{B1}, q^A), q^{B2}(p^{B2}, q^A)\right)\right)$ and $q^{B2}\left(p^{B2}, q^A\left(p^A, q^{B1}(p^{B1}, q^A), q^{B2}(p^{B2}, q^A)\right)\right)$.

$$(p^A - c^A) = -p^A \frac{1}{\varepsilon^A_{own}} - (p^{B1} - c^{B1})\varepsilon^A_{crossB1}\frac{1}{\varepsilon^A_{own}}\frac{q^{B1}}{q^A} - (p^{B2} - c^{B2})\varepsilon^A_{crossB2}\frac{1}{\varepsilon^A_{own}}\frac{q^{B2}}{q^A} \quad (44)$$

for group *A* mark-up,

$$(p^{B1} - c^{B1}) = -p^{B1} \frac{1}{\varepsilon^{B1}_{own}} - (p^A - c^A)\varepsilon^{B1}_{crossA}\frac{1}{\varepsilon^{B1}_{own}}\frac{q^A}{q^{B1}} - (p^{B2} - c^{B2})\varepsilon^{B1}_{crossB2}\frac{1}{\varepsilon^{B1}_{own}}\frac{q^{B2}}{q^{B1}} \quad (45)$$

for group *B1* mark-up, and

$$(p^{B2} - c^{B2}) = -p^{B2} \frac{1}{\varepsilon^{B2}_{own}} - (p^A - c^A)\varepsilon^{B2}_{crossA}\frac{1}{\varepsilon^{B2}_{own}}\frac{q^A}{q^{B2}} - (p^{B1} - c^{B1})\varepsilon^{B2}_{crossB1}\frac{1}{\varepsilon^{B2}_{own}}\frac{q^{B1}}{q^{B2}} \quad (46)$$

for group *B2* mark-up.

Thus, the platform operator has to take into account the own- and cross-price elasticities for all of the sides when setting the profit maximizing price structure.

According to equations (44), (45) and (46), profit maximizing price structure is listed in Table 4 below. The mark-up for group *B1*, the high flexibility agents, is clearly the lower than the mark-up for group *B2*, the low flexibility agents. It is optimal for the platform to attract many *B1* agents into the platform as this makes the platform more valuable for the group *A* agents.

Table 4. Platform pricing, three groups.

	group A	**group B1**	**group B2**
platform price	$p^A = 18.354$	$p^{B1} = 13.610$	$p^{B1} = 17.566$
mark-up	$p^A - c^A = 8.354$	$p^{B1} - c^{B1} = 3.610$	$p^{B2} - c^{B2} = 7.566$

It follows that, as Table 5 illustrates, the pricing structure leads to more group *B1* agents to join the platform than *B2* agents.

Table 5. Platform market share, three groups.

	group A	group B1	group B2
share [%]	S^A =0.270	$S^{B1} = 0.306$	$S^{B2} = 0.228$
group size	$q^A = S^A M^A = 270$	$q^{B1} = S^{B1} M^{B1} = 153$	$q^{B2} = S^{B2} M^{B2} = 114$

Table 6 lists the own- and cross-price elasticities. As can be seen by comparing the own-price elasticity points ($\varepsilon_{own}^{A} = -1.756$, $\varepsilon_{own}^{B1} = -1.152$ and $\varepsilon_{own}^{B2} = -1.479$), the platform operator sets prices such that the price sensitivity for group *B1* (the most valued group) is lowest.

Table 6. Own- and cross-price elasticities, three groups.

	group A	group B1	group B2
price, group A	$\varepsilon_{own}^{A} = \frac{\partial q^A}{\partial p^A}\frac{p^A}{q^A} = -1.756$	$\varepsilon_{crossA}^{B1} = \frac{\partial q^{B1}}{\partial p^A}\frac{p^A}{q^{B1}} = -0.552$	$\varepsilon_{crossA}^{B2} = \frac{\partial q^{B2}}{\partial p^A}\frac{p^A}{q^{B2}} = -0.296$
price, group B1	$\varepsilon_{crossB1}^{A} = \frac{\partial q^A}{\partial p^{B1}}\frac{p^{B1}}{q^A} = -0.685$	$\varepsilon_{own}^{B1} = \frac{\partial q^{B1}}{\partial p^{B1}}\frac{p^{B1}}{q^{B1}} = -1.152$	$\varepsilon_{crossB1}^{B2} = \frac{\partial q^{B2}}{\partial p^{B1}}\frac{p^{B1}}{q^{B2}} = -0.111$
price, group B2	$\varepsilon_{crossB2}^{A} = \frac{\partial q^A}{\partial p^{B2}}\frac{p^{B2}}{q^A} = -0.731$	$\varepsilon_{crossB2}^{B1} = \frac{\partial q^{B1}}{\partial p^{B2}}\frac{p^{B2}}{q^{B1}} = -0.230$	$\varepsilon_{own}^{B2} = \frac{\partial q^{B2}}{\partial p^{B2}}\frac{p^{B2}}{q^{B2}} = -1.479$

Noteworthy is that all of the elasticities non-zero. This implies that although the quantities joining the platform for groups *B1* and *B2* are directly a function of own price and quantity of group *A* agents, $q^{B1}(p^{B1}, q^A)$ and $q^{B2}(p^{B2}, q^A)$, the platform operator has to take into account all of the cross network effects in its pricing decision.

6 Conclusions

Electricity markets are in transition towards systems with larger shares of renewable energy sources. However, the variable nature of renewables like wind and solar power causes new challenges for the security of supply. One solution to the challenge is a larger utilization of electricity consumers as flexibility providing group. With the adoption of smart metering, the consumer involvement on a large scale becomes possible. Given these drivers, this report presents the idea of two-sided platform market for flexibility. The approach is general. Based on economic theory, we go through the main insights regarding to optimal pricing structure. Additionally, an illustrative example consisting of two groups, industry (in need of balancing services) and households (able to provide balancing services), interacting via flexibility platform is presented.

Essentially, platform operator sets the price structure, i.e. prices for both groups, and this decision determines the participation rates for the two sides and the platform profits. Because of the cross network effect the decision is not straightforward as the decision to join the platform for one side is dependent on the participation rate of the other side. Thus, by lowering/rising the price for group *A*, the participation rate of group *A* increases/decreases. This increases/decreases the participation rate of group *B*, which again increases/decreases the participation rate of group *A*, etc. This feedback loop implies that in two-sided markets it is highly important to get the pricing right such that both sides get on board.

The right price structure may thus involve a highly asymmetric price structure. Platform operator sets the price for one side according to the group members' price sensitivity and the value an additional group member brings to the other side group members. With strong cross network effect, the platform price for the valuable group members can be clearly below marginal costs, and even below zero.

In the illustrative example, pricing for the two groups differs as the cross network parameters are set differently for the two groups (see Figure 4). Platform mark-up for group *A* is higher, as group *A* members' utility for an additional group *B* member is higher than group *B* members' utility for an additional group *A* member. In addition, when group *B* was divided into two distinct sub-groups, high flexibility sub-group *B1* and low flexibility sub-group *B2*, prices for flexibility providing groups diverge. High flexibility sub-group *B1* is more valuable group and members are attracted with lower mark-up than the low flexibility sub-group *B2* members.

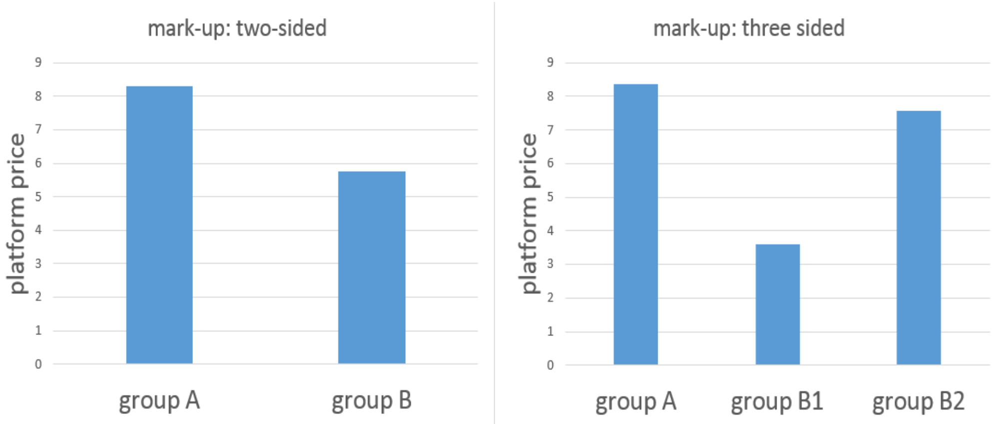

Figure 4 Platform pricing example.

A possible platform operator might be one of the traditional electricity market actors, e.g. DSO, supplier or retailer. The benefit that the incumbent actors have is that the customers and data are readily available. However, in multi-sided markets envelopment is a common strategy and this implies that new entrants may hop into the market by utilizing the networks they have based on the businesses in other industries[7].

[7] One possible example is the acquisition of Nest, the provider of learning thermostat, by Google. Google might combine this technology and the data base collected from the other markets and hop into household energy service market.

References

Areeda, P. and Turner, D. (1975). Predatory pricing and related practices under Section 2 of the Sherman Act. *Harvard Law Review*, 88(4), 697-733.

Argentesi, E. and Filistrucchi, L. (2007). Estimating market power in a two-sided market: The case of newspapers. *Journal of Applied Econometrics 22: 1247-1266.*

Armstrong, M. (2006). Competition in two-sided markets. *RAND Journal of Economics*, Vol. 37, No. 3., Autumn 2006.

Behringer, S. and Filistrucchi, L. (2015). Areeda-Turner in two-sided markets. *Review of Industrial Organization*, 46:287-306.

Eisenman, T., Parker, G,. Van Alstyne, M.W. (2006). Strategies for two-sided markets. *Harvard Business Review*, October 2006.

Evans, D. S. (2011). Platform economics: Essays on multi-sided businesses. Electronic copy available at: http://ssrn.com/abstract=1974020

European Commission (2014a), Cost-benefit analyses & state of play of smart metering deployment in the EU-27, Commission staff working document. Available at: https://ec.europa.eu/energy/en/topics/markets-and-consumers/smart-grids-and-meters

Filistrucchi, L. and Klein, T. J. (2013). Price competition in two-sided markets with heterogemeous counsumers and network effects. *NET Institute working paper* #13-20.

Hagiu, A. and Halaburda, H. (2014). Information and two-sided platform profits. *Harvard Business School Working Paper*, 12-045.

Hirth, L., Ueckerdt, F. and Enedhofer, O. (2015). Integration costs revisited – An economic framework for wind and solar variability. *Renewable energy*, 74: 925-939.

Ikäheimo, J., Evens, C. and Kärkkäinen, S. (2010). DER Aggregator Business: the Finnish Case. Research Report VTT-R-06961-09.

NordREG (2014), Recommendations on Common Nordic Metering Methods, Report 2/2014.

Leiva, J., Palacios, A. and Aguado, J. A. (2016). Smart metering trends, implications and necessities: A policy review. *Renewable and Sustainable Energy Reviews*, 55:227-233.

Rochet, J.-C. and Tirole, J. (2003). Platform competition on two-sided markets. *Journal of the European Economic Association*, Vol. 1., pp. 990-1029.

Rysman, M. (2009). The economics of two-sided markets. *Journal of Economic Perspectives*, Vol. 23(3): 125-143.

Train, K. (2009). Discrete Choice Methods with Simulation, Second Edition. Cambridge University Press.

Weber, C. (2010). Adequate intraday market design to enable the integration of wind energy into the European power systems. *Energy Policy*, 38:3155-3163.

Weiller, C. M. and Pollit, M. G. (2013). Platform markets and energy services. *Cambridge working Papers in Economics.* CWPE 1361 & EPRG 1334

Weyl, E. G. (2010). A price theory of multi-sided platforms. *American Economic Review*, September 2010: 1642-1672.

Wright, J. (2004). One-sided logic in two-sided markets. *Review of network economics*, Vol. 3, Issue 1.

APPENDIX. LOGIT TWO-SIDED MARKET: MONOPOLY PROFIT MAXIMIZATION FIRST ORDER CONDITIONS

The first order conditions derived in Chapter 4 equations (29) and (30) require the calculation of derivatives of both demands q^A and q^B with respect to both prices p^A and p^B. For the optimal price structure solution, implicit function theorem is used.

Quantity vector $q = q(p,q)$ implies that $q(p,q) - q = 0$, and the total derivative is thus

$$\frac{\partial(q(p,q)-q)}{\partial p'}dp + \frac{\partial(q(p,q)-q)}{\partial q'}dq = 0 \tag{A1}$$

Here,

$$\frac{\partial(q(p,q)-q)}{\partial p'} = \begin{bmatrix} \partial q^A(p^A,q^B)/\partial p^A & \partial q^A(p^A,q^B)/\partial p^B \\ \partial q^B(p^B,q^A)/\partial p^A & \partial q^B(p^B,q^A)/\partial p^B \end{bmatrix} = \begin{bmatrix} \partial q^A(p^A,q^B)/\partial p^A & 0 \\ 0 & \partial q^B(p^B,q^A)/\partial p^B \end{bmatrix} \tag{A2}$$

and

$$\frac{\partial(q(p,q)-q)}{\partial q'} = \begin{bmatrix} \partial q^A(p^A,q^B)/\partial q^A & \partial q^A(p^A,q^B)/\partial q^B \\ \partial q^B(p^B,q^A)/\partial q^A & \partial q^B(p^B,q^A)/\partial q^B \end{bmatrix} - \begin{bmatrix} \partial q^A/\partial q^A & \partial q^A/\partial q^B \\ \partial q^B/\partial q^A & \partial q^B/\partial q^B \end{bmatrix} = \begin{bmatrix} -1 & \partial q^A(p^A,q^B)/\partial q^B \\ \partial q^B(p^B,q^A)/\partial q^A & -1 \end{bmatrix} \tag{A3}$$

Consequently, one can solve the derivatives of reduced form demands $\tilde{q}^A(p^A, p^B)$ and $\tilde{q}^B(p^A, p^B)$ as

$$\frac{\partial \tilde{q}'}{\partial p'} = -\begin{bmatrix} -1 & \partial q^A(p^A, q^B)/\partial q^B \\ \partial q^B(p^B, q^A)/\partial q^A & -1 \end{bmatrix}^{-1} \begin{bmatrix} \partial q^A(p^A, q^B)/\partial p^A & 0 \\ 0 & \partial q^B(p^B, q^A)/\partial p^B \end{bmatrix} \quad \text{(A4)}$$

Derivatives of the choice probabilities for group *i*

$$q^i(p^i, q^j) = S^i M^i = \frac{\exp(\alpha^i p^i + \beta^i q^j)}{1 + \exp(\alpha^i p^i + \beta^i q^j)} M^i \quad \text{(A5)}$$

can be written as

$$\frac{\partial q^i(p^i, q^j)}{\partial p^i} =$$

$$\alpha^i \frac{\exp(\alpha^i p^i + \beta^i q^j)}{1 + \exp(\alpha^i p^i + \beta^i q^j)} M^i - \alpha^i \frac{\exp(\alpha^i p^i + \beta^i q^j)}{\left(1 + \exp(\alpha^i p^i + \beta^i q^j)\right)^2} \exp(\alpha^i p^i + \beta^i q^j) M^i = \alpha^i \left(S^i - {S^i}^2\right) M^i = \alpha^i S^i (1 - S^i) M^i \quad \text{(A6)}$$

and

$$\frac{\partial q^i(p^i, q^j)}{\partial q^i} =$$

$$\beta^i \frac{\exp(\alpha^i p^i + \beta^i q^j)}{1 + \exp(\alpha^i p^i + \beta^i q^j)} M^i - \beta^i \frac{\exp(\alpha^i p^i + \beta^i q^j)}{(1 + \exp(\alpha^i p^i + \beta^i q^j))^2} \exp(\alpha^i p^i + \beta^i q^j) M^i =$$

$$\beta^i \left(S^i - {S^i}^2\right) M^i = \beta^i S^i (1 - S^i) M^i \quad \text{(A7)}$$