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Potential of Demand Response in Mitigating the Reserve Requirements of HV Grid

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ABSTRACT of MASTER'S THESIS

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Due to high electricity load growth there is requirement of enhancement of power system network capacity. However, additional capacity requires huge investment. These investments correspondingly increase cost of electricity on customers. To sustain in competitive electricity market, high network efficiency is also necessary. Therefore, there is need to find a way to utilize already kept reserve capacity in the network. Can Demand Response and Electrical Vehicles, Smart Grid features, be utilized to mitigate the reserve capacity requirement?

To find the potential of DR in mitigating the reserve requirements, analysis is conducted in the thesis. Network outage cost is calculated considering different load growths without investing into network. Then decrease in outage cost due to DR in same network is computed. The difference expresses the required potential of DR.

The results of various case studies show that EVs are not able to decrease reserve requirement of grid mainly because their availability at required time is very low. DR potential is also not convincing. Even for low load growth, huge DR resources are required to mitigate the reserve capacity requirement. Study results can be exercised to delay investment in capacity for low growth after comparing with investment cost required. Further evaluation of Potential of DR along with Distributed Energy Resources (DER) is needed.

Keywords: Demand Response, Electric Vehicle, Reserve Requirement, HV Grid, MV Grid

DEDICATION

To Chaudhry Zaheer Sadiq Chattha, for his guidance on how to live simple life with positive frame of mind.

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LIST OF SYMBOLS AND ABBREVIATIONS

a	Year
a_{ij}	Transition rate from state 'i' to state 'j' (Nos. per hour).
A	Transition rate matrix of Markov Model.
c	Contingency counter or contingency variable.
CIC1	Customer Interruption Cost Parameter 1 (€/kW/fault).
CIC2	Customer Interruption Cost Parameter 2 (€/kWh).
C_{DR}	Capacity of Demand Response (%)
DR	Demand Response
DER	Distributed Energy Resources
DLC	Direct Load Control
$E(\tilde{T})$	Mean sojourn time, time spent in any state before transition to next state.
EV	Electric Vehicle
EVs	Electric Vehicles
€	Euros
f	Function of
F1	MV feeder number 1.
F2	MV feeder number 2.
F3	MV feeder number 3.
F4	MV feeder number 4.

F5	MV feeder number 5.
F6	MV feeder number 6.
F7	MV feeder number 7.
F8	MV feeder number 8.
h	Hour or hours
HV	High Voltage
i	State variable
int	Interruption
I & C	Interruptible & Curtailable Load
IEEE	Institute of Electrical and Electronics Engineers
ISO	Independent System Operator
km	Kilometer
kV	Kilovolt
kW	Kilowatt
kWh	Kilowatt-hour
L	Load at load point (kW)
L_{DR}	Decrease in load due to DR
LC	Load Curtailment
LD	Load disconnected in any state (kW).
LP	Load Point
m	Number of load points in the network.

MV	Medium Voltage
MW	Megawatt
Nos.	Numbers
NR	Number of available reserves.
occ	Occurrence
OC	Outage Cost of complete network (€/a).
OC_i	Outage cost for network in state ‘i’ (€).
OC_t	Outage cost for network considering faults at hour ‘t’ (€).
OD_x	Outage duration of load point ‘x’ per year (h/a).
$(OD_x)_i$	Outage duration of load point ‘x’ in state ‘i’ (h).
$(OD_x)_t$	Outage duration of load point ‘x’ considering faults at hour ‘t’ (h).
OF_x	Outage frequency of load point ‘x’ per year (int / a).
$(OF_x)_c$	Outage frequency of load point ‘x’ during contingency ‘c’.
$(OF_x)_i$	Outage frequency of load point ‘x’ in state ‘i’. If a contingency has multiple states in Markov Model then it is considered only once.
$(OF_x)_t$	Outage frequency of load point ‘x’ considering faults at hour ‘t’.
OP_x	Outage power of load point ‘x’ (kW).
$(OP_x)_i$	Outage power of load point ‘x’ in state ‘i’ (kW).
p.u.	per unit

P	Probability matrix for Markov Model of whole network.
P_i	Probability value of system in state ‘i’.
$P_{ij}(t)$	Probability of system in state ‘j’ at hour ‘t’ if present state is ‘i’.
r	Number of states in Markov Model of whole network excluding state ‘0’.
$r + 1$	Total number of states in Markov Model of whole network.
R_1	Reserve state 1.
R_2	Reserve state 2.
std.	Standard
SS	Substation
t	Hour counter or hour variable.
$\mathrm{t_{CB}}$	Switching time for circuit breaker (including fault detection and isolation).
$\mathrm{t_{DR}}$	DR activation time.
$\mathrm{t_{DNR}}$	Distribution network rearrangement time (h).
$\mathrm{t_4}$	Time span required to reach state ‘4’ from state ‘1’, through state ‘3’ (h).
T.L.	Transmission Line
$\mathrm{T_{DR}}$	Demand postponement time without customer interruption cost (hours per day)
T_{ij}	Time spent in state ‘i’ before transition to state ‘j’.
$\mathrm{T_{LC}}$	Time required to curtail load (h).

T_r	Repair time for component (h).
T1	HV/MV transformer number 1.
T2	HV/MV transformer number 2.
T3	HV/MV transformer number 3.
T4	HV/MV transformer number 4.
α_i	Rate of transition to state ‘i’.
Θ_j	Mean duration of visit of state ‘j’ (h).
v_j	Visit frequency of state ‘j’.
v_j^{dep}	Frequency of departure from state ‘j’.
v_j^{arr}	Frequency of arrival to state ‘j’.
λ_C	Failure rate of component.
μ	Transition rate from one state to another.
μ_{10}	Transition rate from state ‘1’ to state ‘0’.
μ_{12}	Transition rate from state ‘1’ to state ‘2’.
μ_{13}	Transition rate from state ‘1’ to state ‘3’.
μ_{20}	Transition rate from state ‘2’ to state ‘0’.
μ_{23}	Transition rate from state ‘2’ to state ‘3’.
μ_{25}	Transition rate from state ‘2’ to state ‘5’.
μ_{30}	Transition rate from state ‘3’ to state ‘0’.
μ_{34}	Transition rate from state ‘3’ to state ‘4’.

μ_{40}	Transition rate from state ‘4’ to state ‘0’.
μ_{46}	Transition rate from state ‘4’ to state ‘6’.
μ_{50}	Transition rate from state ‘5’ to state ‘0’.
μ_{54}	Transition rate from state ‘5’ to state ‘4’.
μ_{60}	Transition rate from state ‘6’ to state ‘0’.
$\mu_{n(n+1)}$	Transition rate from state ‘n’ to state ‘n+1’.
$\mu_{(n+1)0}$	Transition rate from state ‘n+1’ to state ‘0’.
%	Percentage

1 INTRODUCTION

1.1 Research Problem

Nowadays, reliable electricity source is considered basic right. To transport electricity from generation stations to load point power system transmission and distribution infrastructure is required, which makes one of the largest system in the world. The yearly electricity load growth is around 3% worldwide and 2-3% in Europe [5]. Some reserve capacity is always kept in the network which is utilized in minimizing the worse effects of contingencies. A common design of N-1 reliability is used in power system network, which means no loss of supply should be experienced for any single contingency [9].

Due to load growth and limited available capacity of transmission and distribution system there is requirement to enhance the capacity of network. One obvious solution of this problem is to upgrade installation or add new capacity. However, this solution is

1. Expensive as new material is required and right of way for transmission is required.
2. Complex, as right of way need approvals from different authorities.
3. Lengthy
4. May disturb inhabitant and surrounding environment.
5. Cost of electricity increases with increase in investment in the network.

The competitive environment in electricity market has also forced to increase the efficiency of power network already installed. This efficiency can be increased by maximum using the installations. Therefore it is required to search for other possible solutions to cope with increased demand instead of going for huge investments in the network.

With advent in technology, Smart Grid paradigm has developed. One of feature of Smart Grid is Demand Response (DR). DR is utilized to decrease the demand of load in stress situation. This dynamic controllable load can be considered as reserve capacity. Consequently the requirement to keep reserve capacity in network can be mitigated and available reserve can be used for growing load. The conversion of grid to Smart Grid requires investments; to justify investments in DR this thesis evaluates the benefit that can be gained from DR.

The main object of this thesis is to find the potential of DR in mitigating the reserve requirement of grid. Due to faults in the power system network there are corresponding financial losses in form of outage cost. These losses are decreased with increase in redundancy. Potential of DR will be evaluated by considering increased load without investing in network capacity, such that reserve capacity of components is same as initial normal network, then decrease in outage cost due to DR will be calculated. In this thesis outage cost due to contingencies is calculated using reliability assessment method of Markov Model.

1.2 Thesis Organization

Thesis consists of six chapters. After introductory chapter, Chapter 2 provides brief description of Demand Response and Electrical Vehicles. Chapter 3 introduces the reliability assessment method and outage cost. Methodology followed for calculation of outage cost incorporating DR is described in Chapter 4. In Chapter 5 test network details are given and various cases are presented for outage cost comparison. Finally, concluding remarks and future work is presented in Chapter 6.

2 DEMAND RESPONSE (DR)

Demand response is not a new concept; it existed long before the vision of Smart Grids in form of higher tariff in the day and a lower tariff at night. But here, the term demand response is used to denote in a more modern way.

2.1 DR Definition:

Demand Response (DR) is defined as. [19]

"Changes in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized."

The decrease in use of electricity at time of high market price is helpful to reduce the peak demand. Less efficient expensive generators are not required to take into system. Lesser peak demand also delays the investment in power system equipment. Customer response to incentive or utility call is useful during of reserve shortage or contingency.

2.2 Benefits of Demand Response

DR benefits for participant, market and system are mentioned in this section. [28]

2.2.1 Participant Benefits

Financial Benefits

Savings can be made in electricity bill by shifting the load to lower price time. Discounts or benefits can be taken from utility by signing in the different demand response agreements.

Reliability Benefits

Considerable available DR results into lesser unwanted interruption thus reliability of supply increases and higher outage cost is avoided.

2.2.2 Market and System Benefits

Short-Term Market Impacts

Least efficient generators are operated for peak demand. By DR peak demand is reduced. Thus price of electricity in market is reduced.

Long-Term Market Impacts

By reducing peak demand the requirement of additional generation facility, transmission or distribution infrastructure is delayed.

System Reliability Benefits

DR activated during contingencies can act as reserve and reduces load to be interrupted, thus increases overall reliability of system.

2.3 Types of Demand Response

Based on the initiator of demand reduction action there are three type of DR. [20]

2.3.1 Reliability-Based DR Programs

These are also called incentive based programs. DR signal is sent to customer by utility in the stress situation. A customer may have contract with utility of volunteer or compulsory demand reduction in response to DR signal. Direct Load Control (DLC), Interruptible & Curtailable Load (I & C), Emergency Demand Response and Capacity-Market programs lie in this category. DLC loads can be controlled by utility remotely, normally include household appliances e.g. dryer, washer and electric vehicles. I & C load are normally commercial or industrial loads e.g. lighting, process heating, cooling. The response time of DLC loads is faster than I & C loads.

2.3.2 Rate-Based DR Programs

The price of electricity changes dynamically with time such that price is highest for peak hours and lowest for off peak hours. This change in price enforces volunteer reduction in demand from customer. Price of electricity is set prior to actual time of use.

2.3.3 Demand Reduction Bids

Demand reduction bid can be sent by customer to utility with reduction capacity and asked price. Usually large customers participate in this category.

DR can also be classified into **Market DR** and **Physical DR** [26].Market DR is used for real-time pricing via price signals. Physical DR is used for grid management via emergency signals if the grid or parts of its infrastructure (power lines, transformers, substations, etc.) are in a reduced performance due to maintenance or failure. If DR resource is being used as physical DR then it cannot be used as market DR.

2.4 Role of Enabling Technology

For implementation of most of DR programs technology is required. Interval meters with 2 way communications are required to record usage of electricity for each time interval and communicate to utility. Energy-information tools that enable near-real-time access to interval load data, analyze load curtailment performance relative to baseline usage, and provide diagnostics to facility operators on potential loads to target for curtailment. Demand reduction strategies are essential to implement differing high-price or electric system emergency scenarios. Automation of load controls is necessary for control of load under DR. The decrease in cost of advance technology with time has enabled the use of DR. [29]

2.5 DR Research

DR is hot topic in research these days. However, research related to DR has focused on how to **shave off peak demand** of load. Reduced peak load is used to decrease electricity market price, to increase security of supply in case of generation failure and for capacity deferrals.

Few of reviewed papers conclude that, by load curtailment and DR load restored increases and numbers of switch operations are reduced in the distribution system [21]. Nodal and system reliability is improved by DR in deregulated power system [22, 24, and 25]. Demand and load shape can be changed by ISO (Independent

System Operator) policy for running DR programs [23]. By taking off the peak load using DR programs substantial investments can be avoided in local distribution grid [27].

2.6 DR Potential in Finland

Based on survey conducted in 2005, only in large scale industries there is technical potential of about 9% from the peak power of Finland [30]. DR resources will increase as 80% of customers within Distribution Company will have smart meters by 2014.

2.7 Electric Vehicles

There are social, environmental and economic advantages in switching to electricity vehicles [31]. Electric vehicles are often promoted for their environmental performances and are expected to achieve a high share of the commercial market of passenger cars in the future. EV penetration of 100% corresponds to 10% of Finland's annual consumption [35]. EVs act as DR resource when charging unit can be controlled. It is necessary to develop EV interface devices and technology in order to control and schedule the charges [32]. Daily usage and time for connecting to network is different for each user. So, availability of EVs when required as DR resource is low. Charging infrastructure and time of charging of EVs is in the research focus these days [33, 34].

3 RELIABILITY ASSESSMENT AND OUTAGE COST

3.1 Power Quality and Reliability

Consumers use electric power to run their electrical appliances. Power quality is satisfied when it is possible without exceptions. Thus Power quality can be defined as a measure for the ability of the system to let the customers use their electrical equipment. Any event or fault in the power system that prevents the use of appliances when required is lack of power quality. The power quality events can be divided into two groups:

• Interruptions

• Other voltage quality events

The number and severity of power system interruptions are studied in Reliability analysis. Reliability analysis is divided into the field of security analysis and adequacy analysis. Security analysis calculates the number of interruptions due to the transition from one situation to the other. Adequacy analysis looks at interruptions which are due to the outage of one or more primary components in the system. [2]

3.2 Electrical Component Behavior

Failure rate of most of the electrical equipment follow bathtub curve characteristics as shown in Figure 3-1. Failure rate is high for newly installed and aged equipment. High failure rate in beginning is due to manufacturing defects, shipment damages and installation errors. During useful life failure rate is constant and can be represented by scalar quantity. After useful life equipment wears out and fault rate increases again. The behavior of equipment during useful life can be represented by exponential distribution [6]

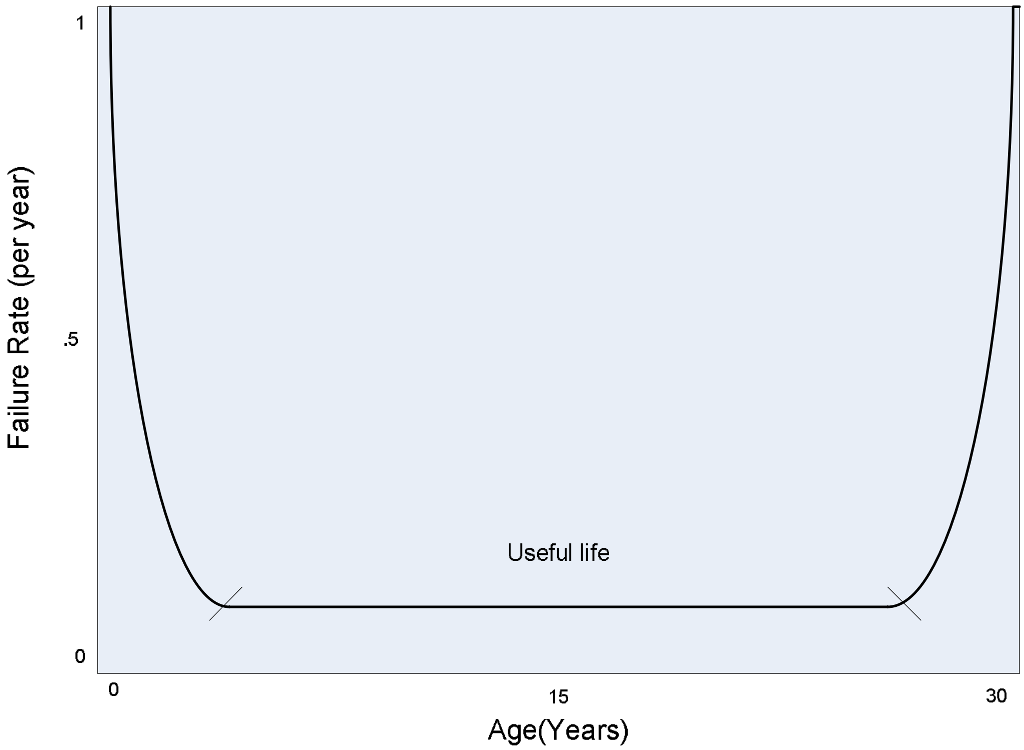

Figure 3-1: ***Bathtub curve, failure rate character of many electrical components.***

A hidden failure (e.g. failure in protection system) may cause multiple component outages. In this thesis hidden failures in protection systems are not taken into account.

3.3 Reliability Assessment Method

Power system behavior is stochastic in nature such as component outages. The development and application of probabilistic techniques for modeling the bulk power system have received considerable attention. In the probabilistic modeling method, uncertainties affecting power system reliability are accounted by using probabilistic techniques. Markov model is widely used; it enables the calculation of probability, frequency, and duration indices of system failures. [1]

3.3.1 Component Model

The transmission and distribution system components can be simply considered of having two operating characteristics either working or failed. Such an operating characteristic can be modeled with a two-state Markov model, as shown in Figure 3-2. [1, 3]

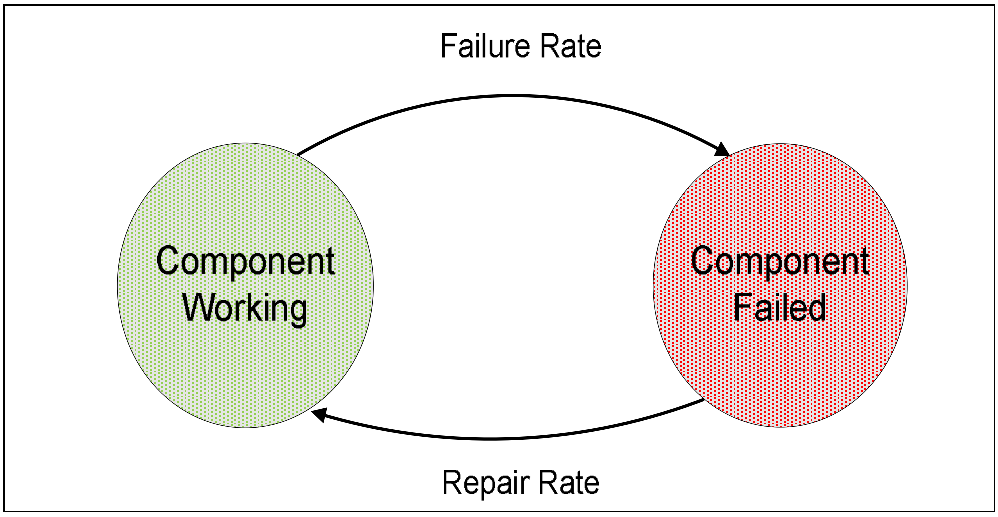

Figure 3-2: 2-State Markov Model

To consider switching after fault three state Markov model is used. Three state Markov model for single component is shown in Figure 3-3. [7, 13, 14, 15]

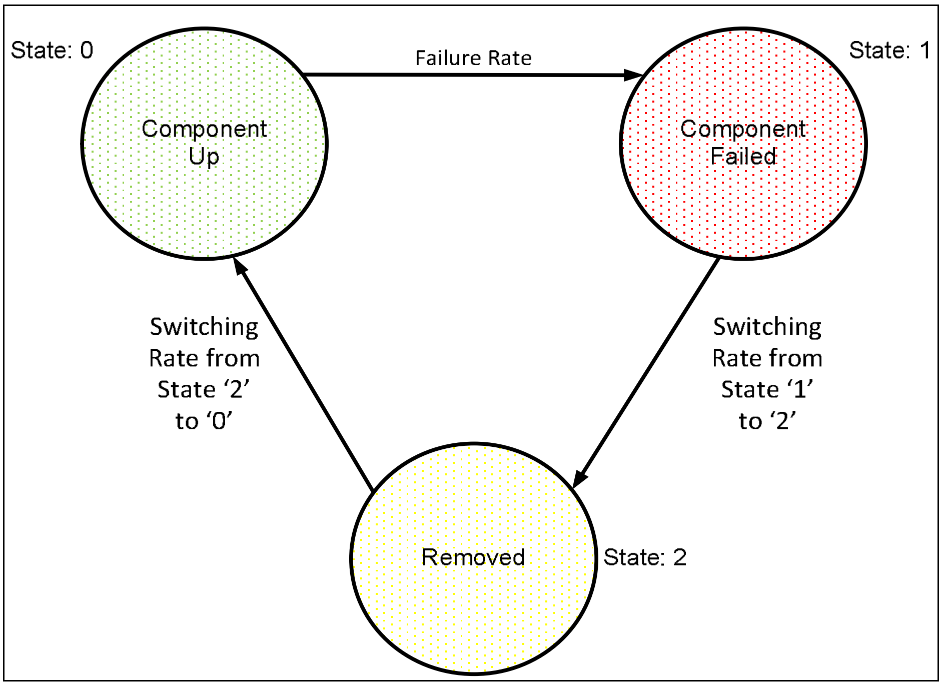

Figure 3-3: 3-State Markov Model.

State ‘0’ is state before fault, state ‘1’ is component failed state and state ‘2’ is state after isolation of faulty component but before repair is complete.

3.3.2 State Enumeration

For a system with n components, number of state possible in two state Markov model will be 2^n.

$For\ n = 100$

$$Possible\ states = 2^{100} \approx 1.3 \times 10^{30} \qquad \textbf{\textit{Eq. 3-1}}$$

If all possible system states (contingencies) are analyzed one by one, the contingency analysis procedure requires too much computational effort and becomes impractical. Therefore, state space reduction technique is required [1]. One method used to reduce state space is by neglecting contingencies with very small probabilities [3]. We can neglect multiple component faults at any time as probability of failure of multi components at a time is low.

$$Possible\ states = C_0^n + C_1^n = C_0^{100} + C_1^{100} = 101 \qquad \textbf{\textit{Eq. 3-2}}$$

3.3.3 Fault Effect Analysis

In this step each failure is analyzed. Switching action is visualized and interruption cost is calculated for disconnected loads [2]. There can be two approaches; adequacy check or security check [3]. In this thesis adequacy shall be checked, that is whether the system is capable of supplying the electric load under the specified contingency without operating constraint violations.

3.4 Reliability Indices

The reliability of power system can be measured by frequency and impact of unwanted events (faults) [4]. There are two types of reliability indices; load point and system. In this section load point indices are considered.

3.4.1 Frequency

The number of interruptions experienced at load point. It is measured in interruptions per year (int /a).

3.4.2 Duration

The duration for which supply is not connected to load. It is measured in hours per year (h /a).

3.4.3 Severity

The amount of load (kW) de-energized due to fault in power system.

3.4.4 Outage Cost

The outage cost consists of two parts ;(1) loss in revenue to utility for energy not supplied (2) Damages to customer in the form of loss in production, waste of under process material, equipment breakdown, man hour loss, etc. Outage cost observed by customer is very high as compared to utility revenue loss. [10]

There are two parameters for customer damage function; (1) to incorporate the effect of frequency of interruptions, here will be called CIC1-customer interruption cost parameter 1(unit of CIC1 is €/kW/fault) (2) to incorporate the effect of duration of interruption, here will be called CIC2- Customer interruption cost parameter2 (unit of CIC2 is €/kWh). The values of these parameters vary widely depending on the customer type e.g. for domestic customer interruption of supply will not affect much, however for industrial customer losses will be very high. Thus corresponding values of parameters will be high for industrial customer compared to domestic customer. [10]

The equation for calculating the outage cost is

$$\begin{aligned} &Outage\ Cost\ for\ a\ load \\ &= Outage\ Power \times (Outage\ Frequency \times CIC1 \\ &\qquad + Outage\ Duration \times CIC2) \end{aligned} \qquad \textbf{\textit{Eq. 3-3}}$$

The outage cost of whole network can be calculated by adding the outage cost of all loads (customers) connected to network

$$Outage\ Cost = \sum Outage\ Cost\ of\ all\ loads \qquad \textbf{\textit{Eq. 3-4}}$$

The advantage of calculating outage cost is that it can be directly used in cost benefit analysis. [4, 10]

4 METHODOLOGY

At first basic Markov model for each network component is drawn. These models are required to be modified to incorporate the effect of DR. Considering model of complete network, outage cost of network is calculated by finding variables (frequency, duration, loads disturbed and outage cost) for each state.

When fault occurs in power system following steps are taken

- Fault detection and clearance by protection
- Fault isolation
- Power restoration by reserve (if available)
- Fault repair
- Re-connection as normal condition

Most of the time in the power system network, reserve capacity for components is available. When a component fails this reserve capacity or reserve component is used to decrease effects of fault. If reserve is able to take all the load disturbed then there will not be any outage after reserve is connected. In case reserve is able to take only partial load then partial load curtailment is required. While transferring load to reserve it is made sure that

- The distribution lines are not overloaded.
- Load on the transformers is within capacity limits.
- Bus-bars are capable of carrying load currents.
- Transmission lines loading limits are not violated.

If reserve connection or supply is not available during the repair, failed component will be out of service, and all customers that cannot be supplied will be interrupted for the duration of the repair [2].

4.1 Basic Models for Network Components

In this section model for each of network components under consideration are drawn. These components are HV sub-transmission line segments, HV busbars, HV/MV transformers, MV busbars and MV cables (distribution feeder segments).

4.1.1 Sub-transmission Line Segment

Normally transmission fulfills criteria of N-1 contingency and fault is automatically cleared by transmission line protection. Therefore single line segment does not result into outage of load. If N-1 criterion is not fulfilled then fault in line segment will result into disconnection of area. Based on the capacity of remaining transmission system there is requirement of curtailment of only portion of load which cannot be supplied. Model for transmission line segments is shown in Figure 4-1.

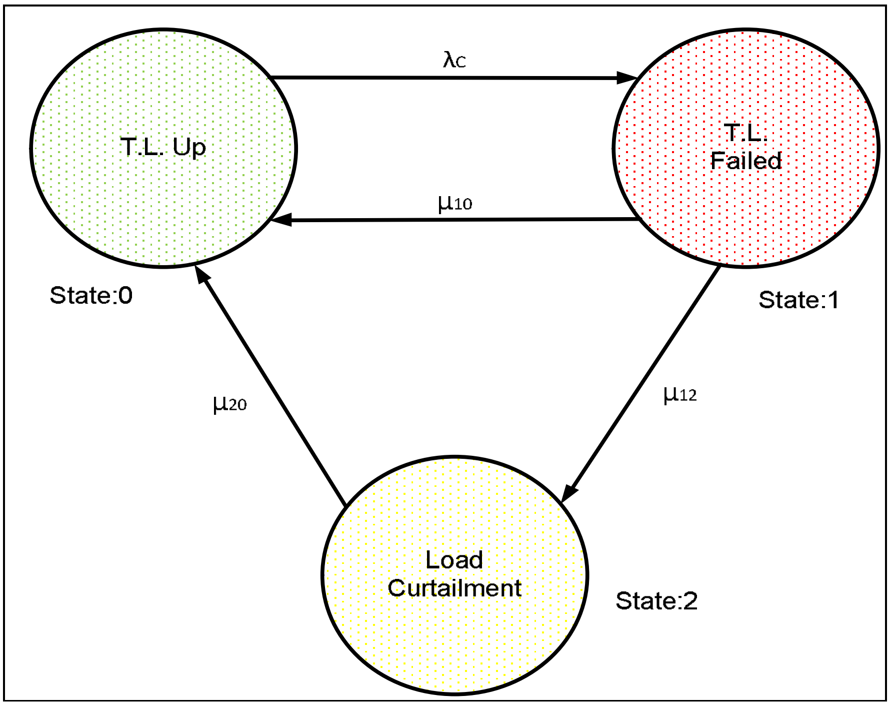

Figure 4-1: Basic model for sub-transmission line segments.

State 0: is normal up state. It represents that all of sub-transmissions line segments are working.

State 1: is failed state. This state shows that one of sub-transmission line segment is in failed state due to fault. Transition from state '0' to state '1' depends on fault rate of lines. If capacity of remaining network is enough to take entire load of network then system will remain in this state till repair of fault. After repair, system goes back to state '0'.

State 2: is load curtailment state. If capacity of remaining network is not enough to take entire load of network then load curtailment is required to avoid thermal heating of lines due to overload. Transition rate from state '1' to state '2' depends on the load curtailment time. When load curtailment is required, sub-transmission lines are allowed to carry load up to short term emergency loading in state '1'. The load curtailed in state '2' will remain unsupplied until repair is complete.

If

λ_C =Failure rate of component (sub-transmission line).

T_r =Time required to repair and reconnect component (sub-transmission line).

T_{LC} =Time required to curtail load.

The transition rates between states are conditional and equal to reciprocal of transition time. Transition time required from any state to state '0' is equal to repair time of component minus time required to reach that from state '1'. Assuming exponential distribution, switching rates from one state to the other are given below.

$$\mu_{10} = \begin{cases} \dfrac{1}{T_r} & If\ LC\ \ not\ required. \\ 0 & else \end{cases} \quad \textbf{Eq. 4-1}$$

$$\mu_{12} = \begin{cases} \dfrac{1}{T_{LC}} & If\ LC\ required. \\ 0 & else \end{cases} \quad \textbf{Eq. 4-2}$$

$$\mu_{20} = \frac{1}{T_r - T_{LC}} \qquad \textbf{Eq. 4-3}$$

4.1.2 Distribution Network Components (HV/MV Transformer, MV Busbars and MV Cables)

The distribution network is usually operated radial. Any fault is distribution network component will produce interruption to loads. There may be multiple reserves available e.g. reserve transformer capacity may be available in the same substation or neighboring substation. During fault of a transformer, if reserve capacity of transformer in the same substation is not enough to carry the entire load then after switching rearrangement partial load can be shifted to neighboring substation transformer. Model for transformers is shown in Figure 4-2. This model is modification of previously built models in research paper [12]. Markov Model design is influenced by switching strategy.

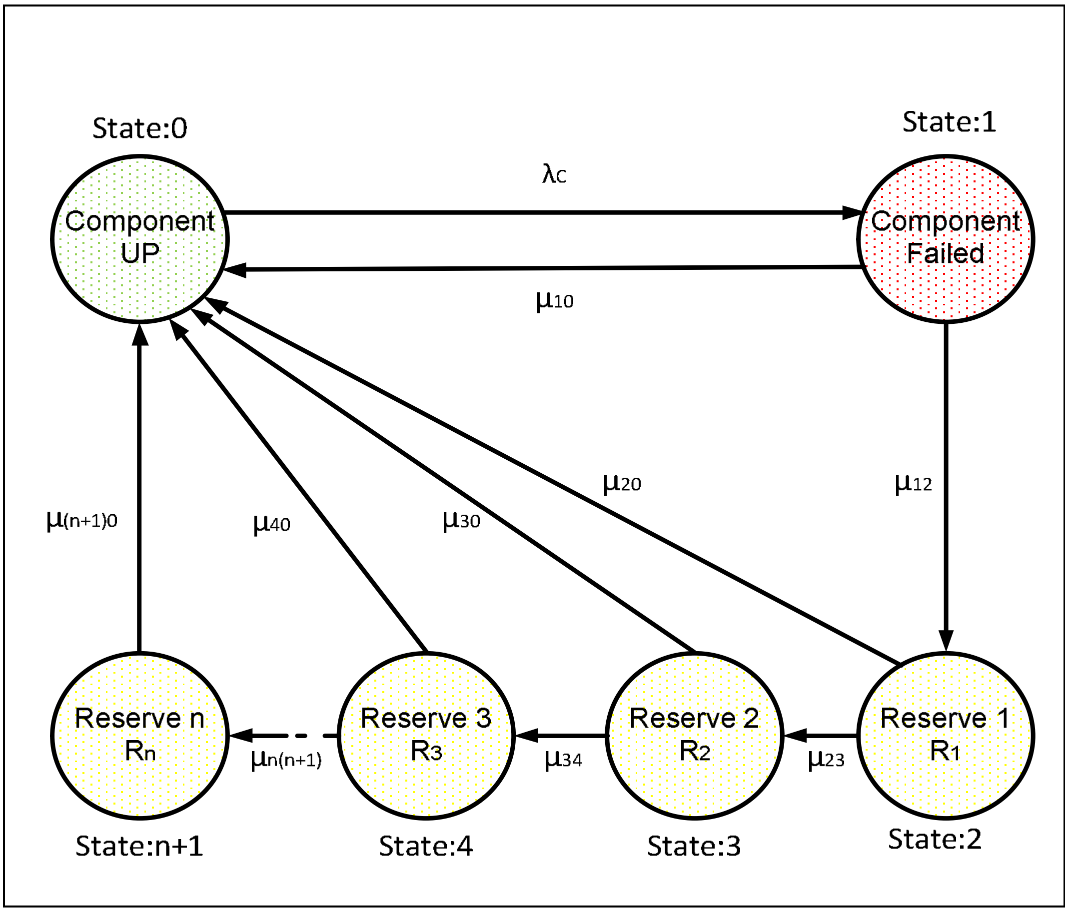

Figure 4-2: Basic model for distribution network components.

State 0: is normal up state. It represents that transformer is working.

State 1: is failed state. This state shows that transformer is in failed state due to fault. The load connected with faulty transformer will be out of supply in this state. Transition from state '0' to state '1' is equal to fault rate of transformer. If reserve is not available then system will remain in this state until repair is completed.

State 2: is first reserve state. The supply of disconnected load is restored in this state. Transition rate from state '1' to state '2' depends on the time required to switch first reserve transformer. If capacity of first reserve transformer is enough to take entire load disconnected then system will remain in this state till repair of fault. After repair, system goes back to state '0'. If capacity of first reserve transformer is not enough to take entire load then partial load will remain unsupplied in this state, and it is required to transfer load to next reserve. Transition from state ‘1’ to state ‘2’ can be achieved in multiple steps e.g. if disconnected feeders are to be energized one by one.

State 3: is second reserve state. The supply of un-energized load in state 2 is restored here. Transition rate from state ‘2’ to state ‘3’ depends on the time required to switch second reserve transformer. If capacity of second reserve transformer is enough to take entire load disconnected then system will remain in this state till repair of fault. After repair, system goes back to state ‘0’. If capacity of second reserve transformer is not enough to take remaining load then partial load will remain unsupplied in this state, and it is required to transfer load to next reserve.

Similarly **State ‘4’** is third reserve state and **state ‘n+1’** is last reserve state.

The transition rate from one state to another is function of time, number of reserves and load disconnected. These transition rates are conditional, number of reserve and load disconnected decide whether rate is zero or some value.

$$\mu = f(T, NR, LD) \qquad \textit{\textbf{Eq. 4-4}}$$

Where

μ is transition rate from one state to another state.

T is time required for switching.

NR is number of reserves available.

LD is amount of load disconnected in any state.

Just like transformers, there can be multiple MV busbars to support system in case of busbar faults. Also more than one feeder may be present for loads of higher priority. Hence, Markov model for MV busbars and cables is same as shown in Figure 4-2 for transformers.

4.1.3 High Voltage Busbars

High voltage busbar configuration can be one of several possible configurations; single bus, sectionalized single bus, breaker-and-a-half, double breaker-double bus and ring bus [11]. Single bus or sectionalized single busbars are normally used on receiving end of power system [36]. Markov model for HV busbars depends on configuration.

For single bus or sectionalized single bus at receiving end of power near load station Markov model will be same as shown in Figure 4-2. Other busbar configurations in transmission network will follow model as shown in Figure 4-1.

4.2 Demand Reduction Due To DR

The decrease in load due to DR (L_{DR}) for duration of repair time of components depends on following factors.

1. Demand Response capacity (C_{DR}in %)
2. Demand postponement time without customer interruption cost (T_{DR}in hours per day)
3. Load at load point (L in kW)
4. Repair time for component (T_r in hours)

The mathematical expression is shown in Eq.4-5.

$$L_{DR} = \begin{cases} C_{DR} \cdot L & If\ T_r \le T_{DR} \\ \dfrac{C_{DR} \cdot L \cdot T_{DR}}{T_r} & If\ \ T_{DR} < T_r < 24 \\ \dfrac{C_{DR} \cdot L \cdot T_{DR}}{24} & If\ \ T_r \ge 24 \end{cases} \qquad \textbf{\textit{Eq. 4-5}}$$

Demand reduction due to DR increases with increase in DR capacity and demand postponement time. Repair time influences if it is between demand postponement time and 24 hours. Where ever required sequential curtailment of DR resources should be done.

If repair time is lesser than or equal to demand postponement time then entire DR resource can be used at same time. The decrease in load demand will be highest in this case. For cases where repair time of component is higher than demand postponement time, entire DR resource cannot be used at same time. To make sure load demand is reduced for repair duration, DR resources are activated sequentially in form of groups. The number of groups is decided by difference in repair time and demand postponement time. Repair time higher than 24 hours will not affect demand reduction as a DR resource is available in a day (24 hours) and after this period same resource can be used again.

4.3 Modified Model Incorporating DR

DR not only decreases the load during contingency but there are other parameters that enforce component model should be different.

1. DR activation time (t_{DR}): Time span required for decreasing load from moment fault observed. Faults on the power system network occur randomly, this parameter gives the idea how fast DR facility can be utilized. Minimum t_{DR} will give maximum benefit.
2. Control level of load: At the level of MV feeder, loads are MV/LV substations. Switching control at this level may result into situation sometimes where load to be interrupted during contingency is same with or without DR e.g. let during contingency there is requirement of load curtailment of 500kW, available decrease in load due to DR activation 100 kW and minimum load that can be curtailed 500kW. In such case DR should not be activated as there is no use of it.

Following sections show the modified models incorporating DR.

4.3.1 Sub-transmission Line Segment with DR

Modified model for sub-transmission line segments is shown in Figure 4-3.

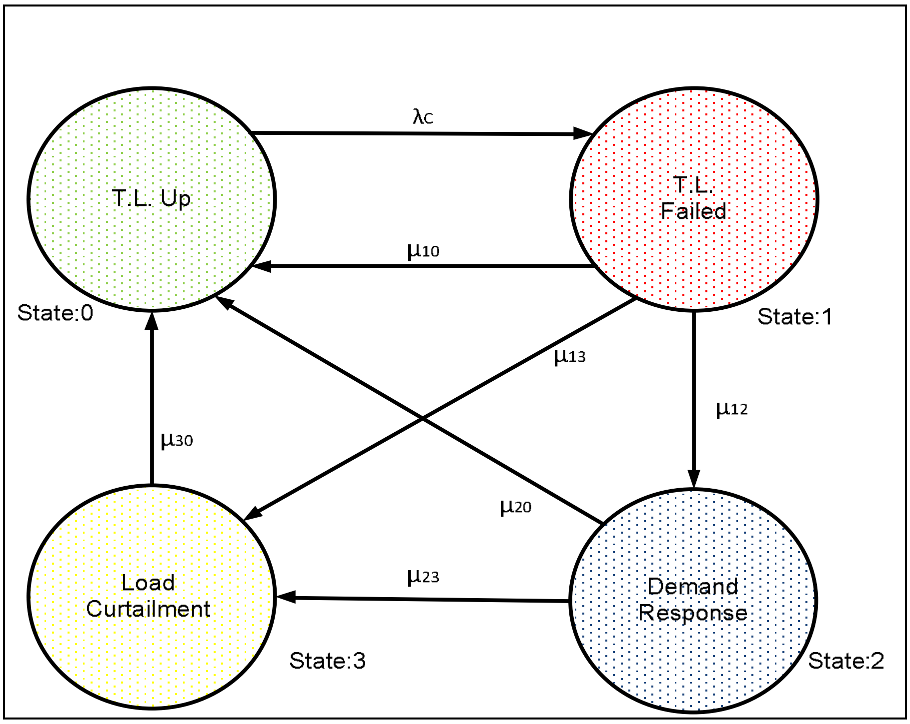

Figure 4-3: Modified model for sub-transmission line segments.

State 0: is normal up state. It represents that all of sub-transmissions line segments are working.

State 1: is failed state. This state shows that one of sub-transmission line segment is in failed state and before load curtailment or activation of DR. Transition from state '0' to state '1' depends on fault rate of lines. If capacity of remaining network is enough to take entire load of network then system will remain in this state till repair of fault. After repair, system goes back to state '0'. If capacity of remaining network is not enough to take entire load then network can be loaded till short term emergency loading capacity in this state before transition to next state.

If

λ_C =Failure rate of component (sub-transmission line).

T_r =Time required to repair and reconnect component (sub-transmission line).

T_{LC} =Time required to curtail load.

t_{DR} =Demand Response (DR) activation time.

$$\mu_{10} = \begin{cases} \dfrac{1}{T_r} & If\ LC\ and\ DR\ activation\ not\ required. \\ 0 & else \end{cases}$$ ***Eq. 4-6***

State 2: is Demand Response (DR) state. This state is visited from state '1' after DR activation time, if DR activation reduces the LC requirement. If LC is not required after DR activation (load at network less than capacity) then system will move to state '0' by completion of repair otherwise state '3' will be visited. Transition time required from state '2' to state '0' is equal to repair time minus time required to reach state '2' from state '1'.

$$\mu_{12} = \begin{cases} \dfrac{1}{t_{DR}} & If\ DR\ reduces\ LC\ requirement. \\ 0 & else \end{cases}$$ ***Eq. 4-7***

$$\mu_{20} = \begin{cases} \dfrac{1}{T_r - t_{DR}} & If\ LC\ not\ required\ after\ DR\ activation. \\ 0 & else \end{cases}$$ ***Eq. 4-8***

State 3: is load curtailment state. Transition to state '3' can be possible either from state '2' or directly from state '1'. If DR does not reduce LC requirement then there is no need to visit state '2', state '3' will be achieved directly from state'1'. After completion of repair system will move to state '0' (Up state).

$$\mu_{13} = \begin{cases} \dfrac{1}{T_{LC}} & If\ DR\ does\ not\ reduce\ LC\ requirement. \\ 0 & else \end{cases}$$ ***Eq. 4-9***

$$\mu_{23} = \begin{cases} \frac{1}{T_{LC}} & If\ LC\ required\ after\ DR\ activation. \\ 0 & else \end{cases} \qquad \textbf{\textit{Eq. 4-10}}$$

$$\mu_{30} = \begin{cases} \frac{1}{T_r - T_{LC}} & If\ DR\ does\ not\ reduce\ LC\ requirement. \\ \frac{1}{T_r - T_{LC} - t_{DR}} & else \end{cases} \qquad \textbf{\textit{Eq. 4-11}}$$

4.3.2 HV/MV Transformer with DR

To simplify, here it is considered that reserve capacity for a transformer may be available in two other transformers, first in same substation and second in neighboring substation. Modified model for HV/MV transformer is shown in Figure 4-4.

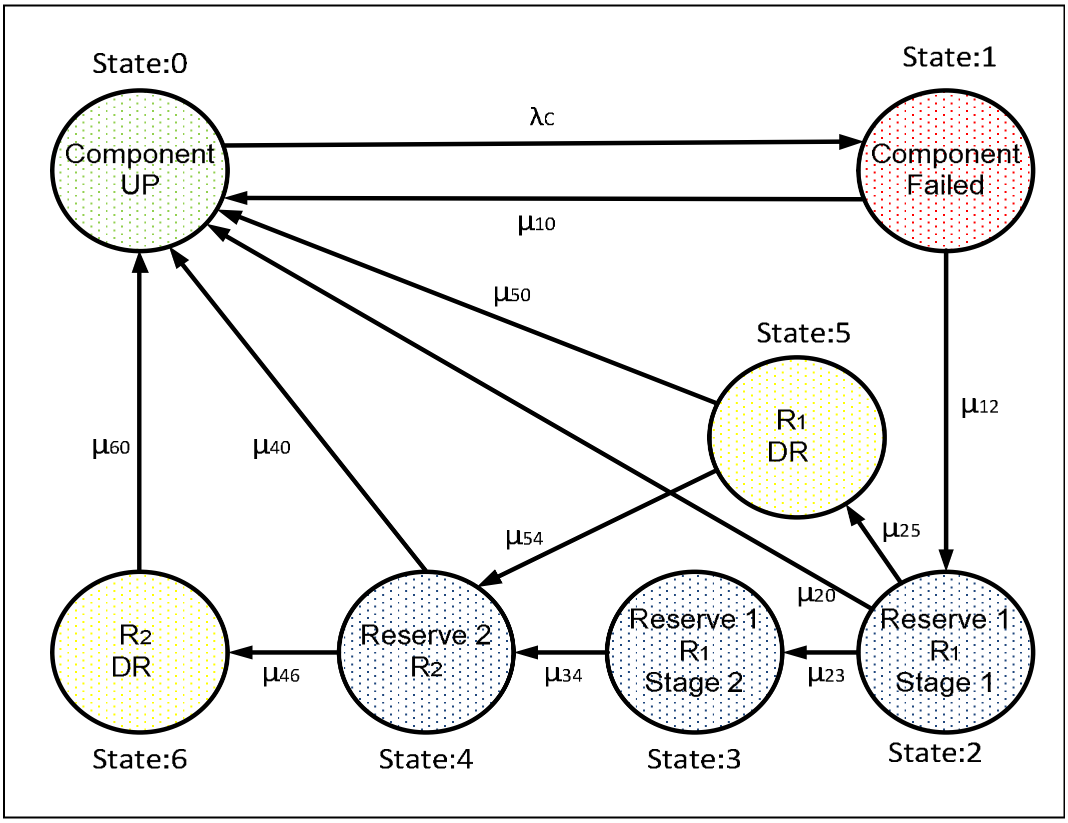

Figure 4-4: Modified model for HV/MV transformer.

State 0: is normal up state. It represents that transform is working.

State 1: is failed state. This state shows that transformer is in failed state due to fault. The load connected with faulty transformer will be out of supply in this state. Transition rate from state '0' to state '1' is equal to fault rate of transformer. If reserve and DR not available, system will remain in this this till completion of repair.

$$\mu_{10} = \begin{cases} \frac{1}{T_r} & If\ DR\ and\ reserve\ not\ available. \\ 0 & else \end{cases} \qquad \textbf{\textit{Eq. 4-12}}$$

State 2: is stage 1 of first reserve state. After circuit breaker switching time (including fault detection and isolation) disturbed feeders are connected to reserve transformer in the same substation. It is made sure that transformer is not loaded more than short term emergency rating. If long term emergency capacity of first reserve transformer is enough to take entire load disconnected then system will remain in this state till repair of fault. After repair, system goes back to state '0'. If long term emergency capacity of first reserve transformer is not enough to take entire load then partial load will remain unsupplied in this state, and it is required to either activate DR or transfer load to next reserve. Transition from state '1' to state '2' can be further divided in multiple steps e.g. if disconnected feeders are to be energized one by one

If

λ_C = Failure rate of component (transformer).

T_r = Time required to repair and reconnect component (transformer).

T_{LC} =Time required to curtail load.

t_{DR} = Demand Response (DR) activation time.

t_{CB} = Circuit breaker switching time (including fault detection and isolation).

t_{DNR} = Distribution network rearrangement time.

R_1 =Reserve 1 state.

R_2 = Reserve 2 state.

$$\mu_{12} = \frac{1}{t_{CB}} \qquad \textbf{\textit{Eq. 4-13}}$$

$$\mu_{20} = \begin{cases} \dfrac{1}{T_r - t_{CB}} If\ DR\ activation\ and\ LC\ not\ required(R_1) \\ 0 \quad else \end{cases} . \qquad \textbf{\textit{Eq. 4-14}}$$

State 3: is stage 2 of first reserve state. If DR activation will not able to reduce LC in first stage of reserve 1 then after load curtailment time this state is achieved. Transformer is loaded not more than long term emergency load rating. As some quantity of load is disconnected in this state thus transition from this state to second reserve state will always happen.

$$\mu_{23} = \begin{cases} \dfrac{1}{T_{LC}} \quad If\ DR\ activation\ \ not\ required, LC\ required\ (R_1) \\ 0 \quad else \end{cases} . \qquad \textbf{\textit{Eq. 4-15}}$$

State 5: is first reserve with demand response state. This state is achieved if DR activation is able to reduce LC in reserve 1. Transformer is loaded not more than long term emergency load rating. If DR activation eliminates LC requirement then after DR activation time state '5' is visited and state '0' is achieved after it. Otherwise this transition need sum of DR activation and load curtailment time and state '4' is visited after it

$$\mu_{25} = \begin{cases} \dfrac{1}{t_{DR}} \quad If\ DR\ activation\ required\ and\ LC\ not\ required(R_1) \\ \dfrac{1}{T_{LC} + t_{DR}}\ elseIf\ DR\ activation\ and\ LC\ both\ required\ (R_1) \\ 0 \quad else \end{cases} \qquad \textbf{\textit{Eq. 4-16}}$$

$$\mu_{50} = \begin{cases} \frac{1}{T_r - t_{CB} - t_{DR}} & If\ DR\ activation\ eliminates\ LC\ requirement(R_1) \\ 0 & else \end{cases} \quad \textbf{Eq. 4-17}$$

State 4: is second reserve state, corresponds to transformer in neighboring substation. The supply of un-energized load in reserve 1 is restored here. This state is visited after network rearrangement time either from state '3' or state '5'. If long term emergency capacity of second reserve transformer is enough to take balance load or DR activation does not reduce LC requirement in reserve 2 then system will remain in this state till repair of fault. If DR activation is required then short term emergency loading can be applied on this transformer.

$$\mu_{34} = \frac{1}{t_{DNR}} \quad \textbf{Eq. 4-18}$$

$$\mu_{54} = \begin{cases} \frac{1}{t_{DNR}} & If\ DR\ activation\ and\ LC\ required(R_1) \\ 0 & else \end{cases} \quad \textbf{Eq. 4-19}$$

$$t_4 = t_{DNR} + T_{LC} + t_{CB} \quad \textbf{Eq. 4-20}$$

$$\mu_{40} = \begin{cases} \frac{1}{T_r - t_4} & If\ DR\ not\ required(R_1\ and\ R_2) \\ \frac{1}{T_r - t_4 - t_{DR}} & elseIf\ DR\ required\ (R_1), not\ required(R_2) \\ 0 & else \end{cases} \quad \textbf{Eq. 4-21}$$

Here t_4 is time required to reach state '4' from state '1' through state '3'. Additional DR activation time needed if state '4' is accessed through state '5'.So, μ_{40} is reciprocal of repair time minus state '4' reach time.

State 6: is second reserve with demand response state. This state is achieved from state '4' if DR activation is able to reduce LC in reserve 2. From here, next state will always state '0', after completion of repair.

$$\mu_{46} = \begin{cases} \dfrac{1}{t_{DR}} \; If\ DR\ activation\ required(R_2) \\ 0 \quad else \end{cases} \qquad \textbf{\textit{Eq. 4-22}}$$

$$\mu_{60} = \begin{cases} \dfrac{1}{T_r - t_4 - t_{DR}} \; If\ DR\ activation\ not\ required(R_1) \\ \dfrac{1}{T_r - t_4 - 2t_{DR}} \quad else \end{cases} \qquad \textbf{\textit{Eq. 4-23}}$$

μ_{60} is reciprocal of repair time minus state '6' reach time.

4.3.3 MV Cables with DR

If more than one cable is connected to load point, first reserve cable is used to supply load during cable contingency. Second reserve cable is used if first reserve also fails during repair time. In this thesis maximum single fault at a time is considered, so, Morkov Model considering DR for MV Cables with single reserve available is drawn in Figure 4-5.

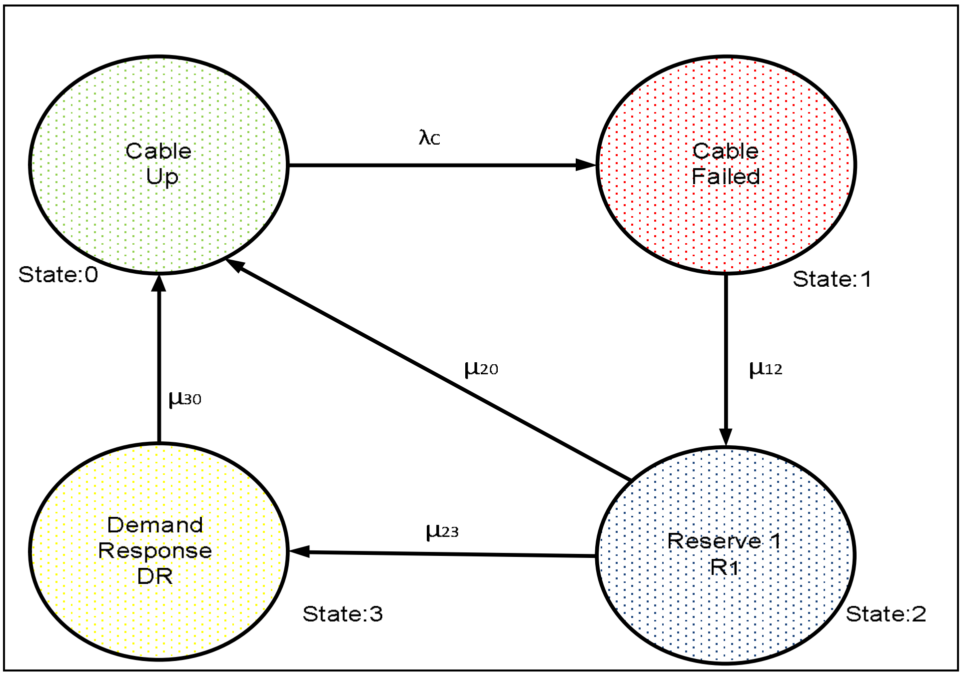

Figure 4-5: Modified model for MV Cables.

State 0: is normal up state. It represents that cable is working.

State 1: is failed state. This state shows that cable is in failed state due to fault. The load connected with faulty cable will be out of supply in this state. Transition rate from state '0' to state '1' is equal to fault rate of cable.

State 2: is reserve state. After manual switching time (including fault detection and isolation) faulty cable section is taken out of system and supply is restored to disturbed loads via healthy section of cable and reserve cable. It is considered that switches at load points are manual. If DR activation is needed to take disturbed load then during this manual switch time DR is activated for load present at reserve cable. It is made sure that cable is not overloaded. If emergency capacity of reserve cable is enough to take entire load disconnected then system will remain in this state till repair of fault. After repair, system goes back to state '0'. If emergency capacity of reserve cable is not enough to take entire load then partial load will remain unsupplied.

State 3: is reserve with demand response state. This state is achieved if DR activation is required in reserve state. DR is activated for Load which was initially disconnected due to fault and energized in state '2'. State '0' is achieved after it. Load disconnected in this state will remain unsupplied till repair.

If

λ_C = Fault rate of component (cable).

T_r = Time required to repair and reconnect component (cable).

t_{ms} = Manual switching time (including detection and reconnection).

t_{DR} = Demand Response (DR) activation time.

Based on above mentioned conditions for each state, transition rates are given in below equations.

$$\mu_{12} = \frac{1}{t_{ms}}$$ ***Eq. 4-24***

$$\mu_{20} = \begin{cases} \frac{1}{T_r - t_{ms}} & If\ DR\ activation\ not\ required. \\ 0 & else \end{cases}$$ ***Eq. 4-25***

$$\mu_{23} = \begin{cases} \frac{1}{t_{DR}} & If\ DR\ activation\ is\ required. \\ 0 & else \end{cases}$$ ***Eq. 4-26***

$$\mu_{30} = \frac{1}{T_r - t_{DR} - t_{ms}}$$ ***Eq. 4-27***

4.3.4 Busbars with DR

Modified model for MV busbars will be same as of HV/MV transformer model. For HV single bus or sectionalized single bus at receiving end of power near load station Markov model will be same as shown in Figure 4-4. Other HV busbar configurations in transmission network will follow model as shown in Figure 4-3.

4.4 Outage Cost Calculation for Complete Network

Flow diagram for calculating reliability indices at load point and total outage cost of network is shown in Figure 4-6.

Module 1: In this module data related to network is obtained. The data may include

- Electrical components types (e.g. Overhead lines, underground cables, transformers and busbars).
- Rating of components (e.g. voltage, current, normal capacity, long term emergency capacity and short term emergency capacity).
- Interconnection of components.
- Fault rates of components.
- Repair time of components.
- Operation procedures.
- Network configuration.
- Load point data (e.g. load pattern, power factor and interruption cost).

Module 2: Faults in power system occur randomly. For hourly varying load outage cost of network depends on the fault instant. So, it is required to calculate the effect of fault considering fault occurrence at each hour. Hour counter or hour variable ‘t’ is initialized here.

Module 3: There are multiple components in power system network and each component is prone to faults. It is required to analyze each contingency in order to calculate outage cost. Contingency counter or contingency variable ‘c’ is initialized here.

Module 4: In this module, variables corresponding to disconnected load point due to contingency are stored; variables contain information whether load is disconnected due to contingency. If reserve or DR is available, variables for load points to be disconnected even after activation of DR or switching of reserve are also calculated.

This module is revisit until load point disconnection for all contingencies has been calculated.

Module 5: In this module, Markov Model for network considering all contingencies at hour ‘t’ is formed. This model is combination of individual component model built in section 4.1 and 4.3. Mathematically Markov Model is presented as a transition rate matrix (A). Transition rate matrix is of order ‘r+1× r+1’.

$$A = \begin{bmatrix} a_{00} & a_{01} & \cdots & a_{0r} \\ a_{10} & a_{11} & \cdots & a_{1r} \\ \vdots & \vdots & \ddots & \vdots \\ a_{r0} & a_{r1} & \cdots & a_{rr} \end{bmatrix} \qquad \textit{\textbf{Eq. 4-28 [8]}}$$

Where $a_{ij}\ for\ i \neq j$ is transition rate from state i to state j and a_{ii} is such that sum of elements in a row is zero.$'r + 1'$ is equal to number of states in model.

Probability of system in state $i(P_i)$ is calculated from these two equations.

$$\boldsymbol{P} \cdot A = 0 \qquad \textit{\textbf{Eq. 4-29 [8]}}$$

$$\sum_{i=0}^{r} P_i = 1 \qquad \textbf{\textit{Eq. 4-30 [8]}}$$

Where

$$\boldsymbol{P} = [P_0 \quad P_1 \quad \cdots \quad P_r] \qquad \textbf{\textit{Eq. 4-31 [8]}}$$

Module 6: Here, visit frequency(v_j) of each state (j) and Mean duration of visit(Θ_j) are calculated from transition rate matrix(A) and probability matrix(P). Derivation of these is explained in Appendix.

$$v_j = \sum_{k=0, k \neq j}^{r} P_k \, a_{kj} \qquad \textbf{\textit{Eq. 4-32 [8]}}$$

$$\Theta_j = \frac{P_j}{v_j} \qquad \textbf{\textit{Eq. 4-33 [8]}}$$

Load points disconnected in each state are also evaluated in this module.

Module 7: In this module outage frequency of each load, outage duration of each load and outage cost for faults at hour 't' is calculated.

Outage frequency for a load point 'x' considering fault at instant 't' is sum of outage frequencies for all contingencies at 't'.

$$(OF_x)_t = \sum_{c=1}^{n} (OF_x)_c \times v_c \qquad \textbf{\textit{Eq. 4-34}}$$

Where

c = Contingency counter or contingency variable.

n = Total number of contingencies.

v_c =Visit frequency of contingency 'c' (visit frequency of state corresponding to 'c').

$(OF_x)_c$ = Outage frequency of load point ‘x’ during contingency ‘c’.

$(OF_x)_c = \begin{cases} 1 & If\ load\ x\ is\ disconnected. \\ 0 & else \end{cases}$

$(OF_x)_t$ = Outage frequency of load point ‘x’ considering faults at hour ‘t’.

Outage duration for load point ‘x’ considering fault at instant ‘t’ is sum of outage durations for ‘x’ in all the states of Markov Model.

$$(OD_x)_t = \sum_{i=1}^{r} (OD_x)_i \qquad \textbf{\textit{Eq. 4-35}}$$

Where

i = State counter or state variable.

r = Total number of states in Markov Model excluding state ‘0’.

$(OD_x)_i = \Theta_i$ = Outage duration of load point ‘x’ in state ‘i’ (if load is disconnected) (h).

$(OD_x)_t$ = Outage duration of load point ‘x’ considering faults at hour ‘t’ (h).

Outage cost for network in state ‘i’ is calculated by following equation

$$OC_i = \sum_{x=1}^{m} (OP_x)_i \times ((OF_x)_i \times CIC1 + (OD_x)_i \times CIC2) \qquad \textbf{\textit{Eq. 4-36}}$$

Where

i = State counter or state variable.

x = Load point counter or load point variable.

m = Total number of load points in the network.

$(OP_x)_i$ = Outage power of load point 'x' in state 'i' (kW).

$(OF_x)_i$ = Outage frequency of load point 'x' in state 'i'. If a contingency has multiple stated in Markov Model then it is considered only once.

$(OD_x)_i$ = Outage duration of load point 'x' in state 'i' (h).

$CIC1$ = Customer interruption cost parameter 1 (€/kW/fault)

$CIC2$ = Customer interruption cost parameter 2 (€/kWh)

OC_i = Outage cost for network in state 'i' (€).

Outage cost of network considering faults at instant 't' is sum of outage costs in all states.

$$OC_t = \sum_{i=1}^{r} OC_i \qquad \textit{Eq. 4-37}$$

Where

i = State counter or state variable.

r = Total number of states in Markov Model excluding state '0'.

OC_i = Outage cost for network in state 'i' (€).

OC_t = Outage cost for network considering faults at hour 't' (€).

A year consists of 8760 hours; steps from module 3 to module 7 are repeated 8760 times.

Module 8: Finally results of previous modules are added to calculate outage frequency, outage duration and outage cost for whole network per year.

$$OF_x = \sum_{t=1}^{8760} (OF_x)_t \qquad \textbf{\textit{Eq. 4-38}}$$

$$OD_x = \sum_{t=1}^{8760} (OD_x)_t \qquad \textbf{\textit{Eq. 4-39}}$$

$$OC = \sum_{t=1}^{8760} OC_t \qquad \textbf{\textit{Eq. 4-40}}$$

Where

t = Hour counter or hour variable.

$(OF_x)_t$ = Outage frequency of load point ‘x’ considering faults at hour ‘t’.

OF_x = Outage frequency of load point ‘x’ per year (int / a).

$(OD_x)_t$ = Outage duration of load point ‘x’ considering faults at hour ‘t’ (h).

OD_x = Outage duration of load point ‘x’ per year (h/a).

OC_t = Outage cost for network considering faults at hour ‘t’ (€)

OC = Total outage cost for network per year (€/a).

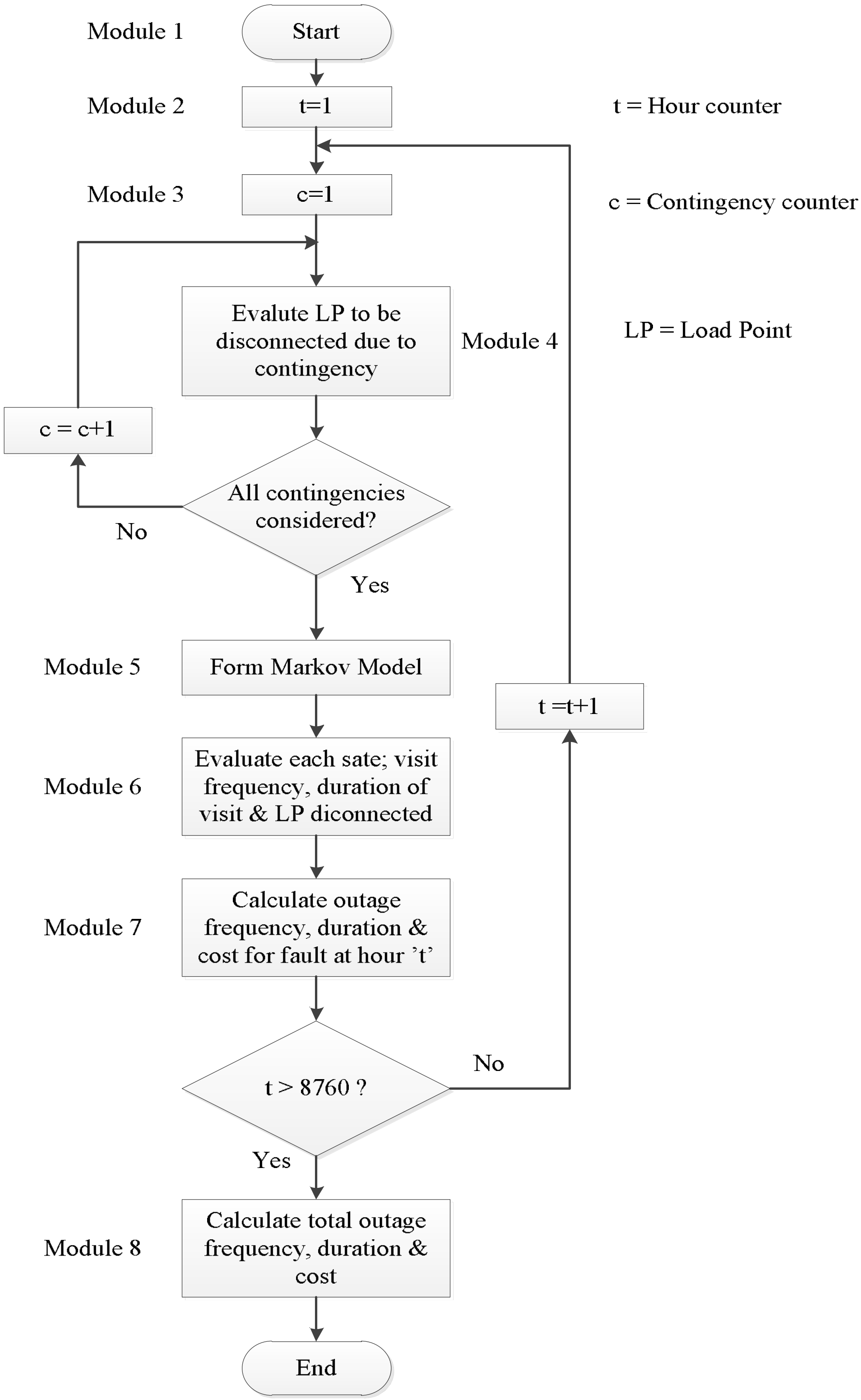

Figure 4-6: Flow diagram for calculating reliability indices and total outage cost.

5 STUDY RESULTS

During thesis, a program has been developed to calculate reliability indices and outage cost due to faults in sub-transmission and primary distribution network. With help of this program, results for different case studies have been produced and are discussed in this chapter.

5.1 Test System

A typical Finish sub-transmission (110kV) and primary distribution (20 kV) network is considered as test system. Single line diagram of test system is shown in Figure 5-1. Overall data for test system is listed in Tables 5-1and 5-2. There are 12 sub-transmission lines to supply power to two HV/MV substations. Each HV/MV substation has two 110/20 kV transformers. These transformers are connected to two MV feeders via MV busbars. Each MV feeder consists of two sections, normally open from midpoint, operated independently.

Components reliability data is considered as mentioned in Table 5-3. [10, 16]

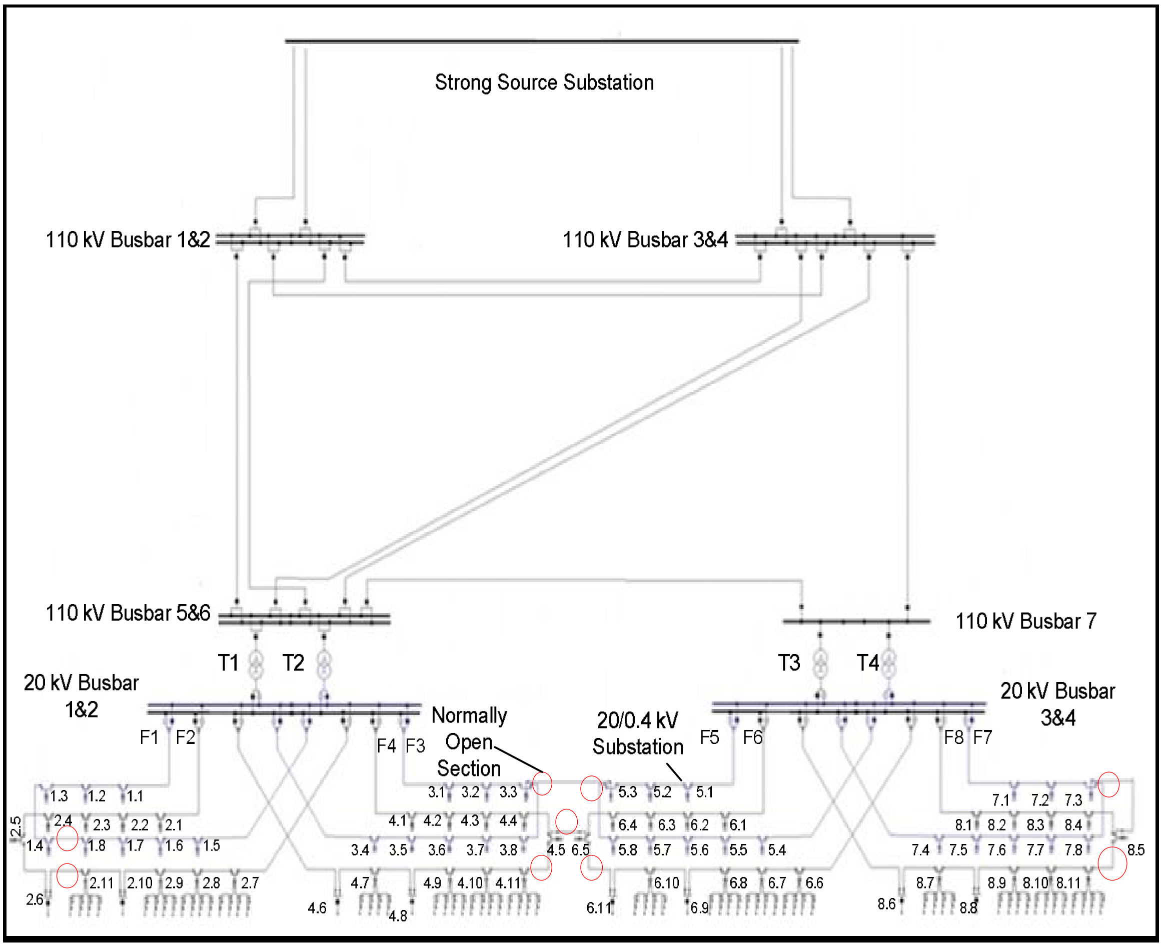

Figure 5-1: Single-line diagram of typical Finish sub-transmission (110kV) and primary distribution (20kV) network which is used as test network.

Table 5-1: Basic data for test network.

Attribute	Value (Nos.)
110 kV Lines	12
110 kV Busbars	7
110/20 kV Transformers	4
20 kV Busbars	4
20 kV Feeders	8

Table 5-2: Basic data for distribution test network.

Distribution Feeder	Number of Distribution Substations	Peak Load (MW)
F1	8	8
F2	11	11
F3	8	8
F4	11	11
F5	8	8
F6	11	11
F7	8	8
F8	11	11
Total	**76**	**76**

Table 5-3: Component reliability data for test network. [10, 16]

Component	Failure Rate	Repair Time (h)
110 kV Line	0,0218 (occ/km-a)	48
110kV Busbars	0,0068 (occ/a)	200
110/20 kV Transformer	0,023 (occ/a)	120
20 kV Busbar	0,0068 (occ/a)	12
20 kV Feeder (Cables)	0,006 (occ/km-a)	10

5.2 Load Profile

The load profile at MV/LV transformers depends on the type of customers connected to it. Here it is considered that two types of consumers are connected to each MV/LV substation. Load of each type of consumer is typical hourly varying load. The load pattern of each MV/LV substation fed through HV/MV transformers T1 and T2 is shown in Figure 5-2. T1 and T2 are supplying power to area where consumers are office, shops and district/oil heating houses. There is not much difference in load demand between summer and winter working week as shown in Figure 5-3.

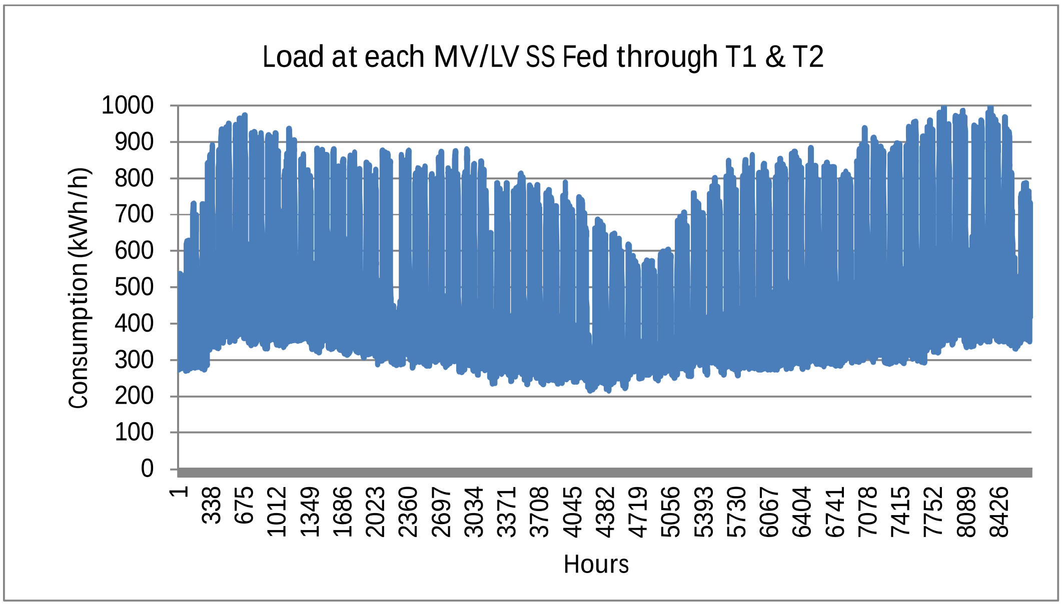

Figure 5-2: Annual Load profile at each MV/LV substation fed through T1 & T2.

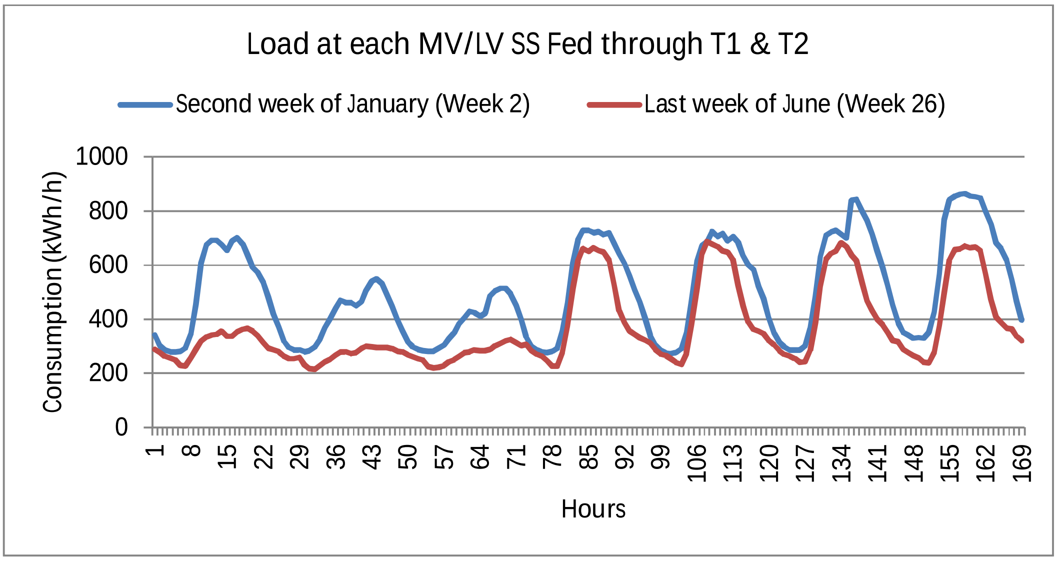

Figure 5-3: Load profile of specific week at each MV/LV substation fed through T1 & T2.

The load pattern of MV/LV substations fed through HV/MV transformers T3 and T4 is shown in Figure 5-4. T3 and T4 are supplying power to area where consumers are two types of houses, with electric and district/oil heating. There is considerable difference in load demand between summer and winter working week as shown in Figure 5-5.

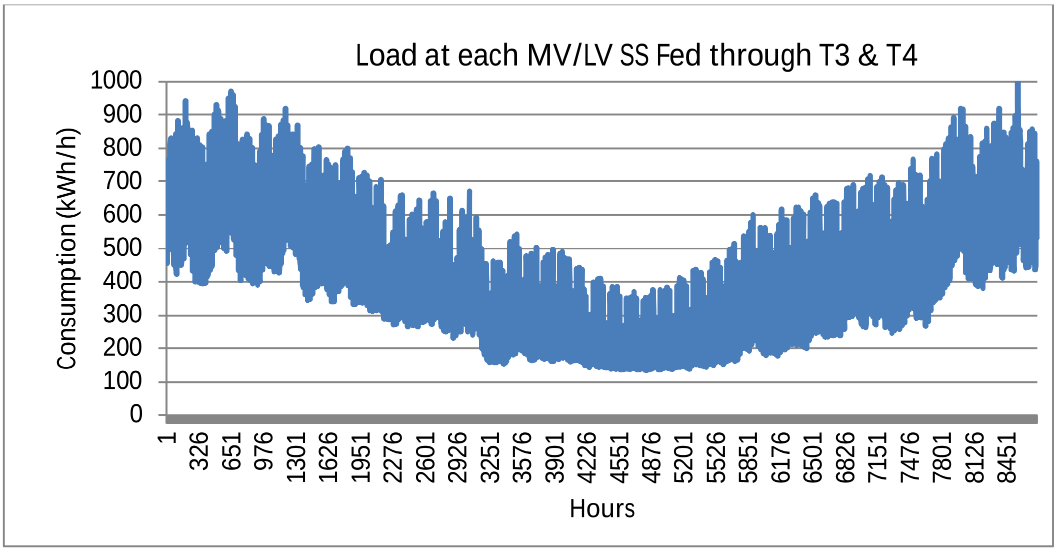

Figure 5-4: Annual Load profile at each MV/LV substation fed through T3 & T4.

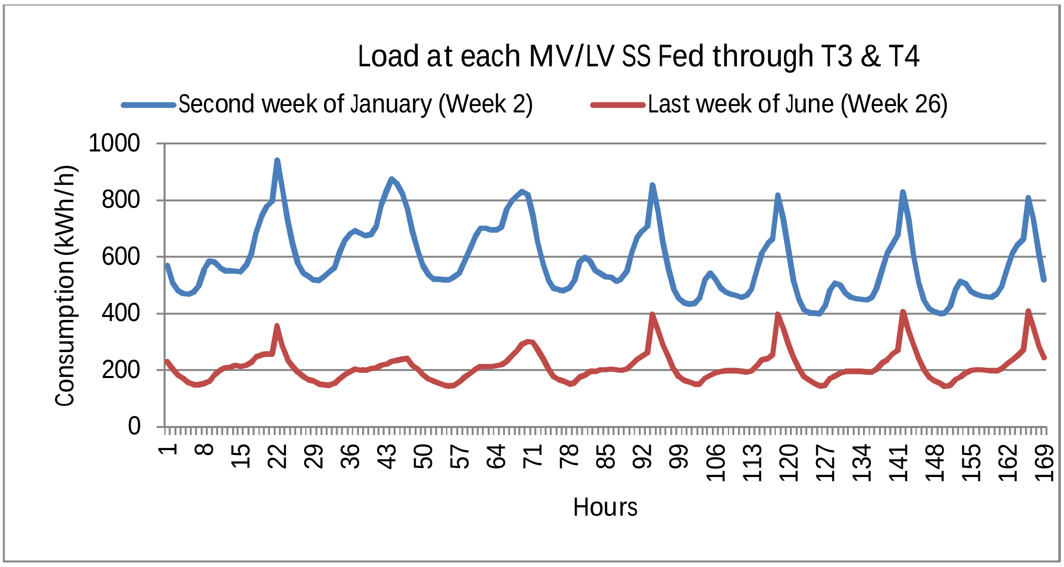

Figure 5-5: Load profile of specific week at each MV/LV substation fed through T3 & T4.

5.3 Analysis Assumptions

For the analysis of test system, following assumptions are made.

- The length of each 110 kV line segment is 10 km.
- The length of each 20 kV feeder segment is 0,5 km.
- The long term emergency capacity of each transformer at present is equal to total load at HV/MV substation at present. In case of failure of one transformer other transformer is able to take all the load of substation.
- Capacity of Feeders F1 is such that it can take load of all (8 Nos.) MV/LV substation on that feeder.
- Capacity of Feeders F2 is such that it can take load of all (11 Nos.) MV/LV substation on that feeder.
- Capacity of Feeders F3 is such that it can take load of all (16 Nos.) MV/LV substation on feeders F3 and F5.
- Capacity of Feeders F4 is such that it can take load of all (22 Nos.) MV/LV substation on feeders F4 and F6.

- Capacity of Feeders F5 is such that it can take load of all (16 Nos.) MV/LV substation on feeders F3 and F5.
- Capacity of Feeders F6 is such that it can take load of all (22 Nos.) MV/LV substation on feeders F4 and F6.
- Capacity of Feeders F7 is such that it can take load of all (19 Nos.) MV/LV substation on feeders F7 and F8.
- Capacity of Feeders F8 is such that it can take load of all (19 Nos.) MV/LV substation on feeders F7 and F8.
- N-1 reliability criterion is satisfied for all types of faults at present.
- Transformers are loaded 60% normally. Long term emergency loading capacity is 120% and short term emergency loading capacity is 150%. Short term emergency loading is used during switching actions.
- Overhead lines long term emergency load capacity is 100 % and short term emergency loading capacity is 110%. Short term emergency loading is used during switching actions.
- Underground cables are never overloaded.
- All the switches in the distribution network are manual, switching time = 0,5 hours.[10]
- Switching time of circuit breaker = 0,0015 hours.
- Distribution network rearrangement time is 3 hours, when load of one HV/MV substation to be shifted to other HV/MV substation in case of capacity constraint due to transformers or busbar failure.
- All the loads in the network have the same value of customer damage function parameters. Parameter1 (CIC1 =) 1€/kW/fault and parameter 2 (CIC2=) 10 €/kWh.
- Electrical Vehicles (EVs) are connected to network 3 h in a day for charging (probability of cars being in the network is 3/24).
- Demand Response (DR) activation time from moment of fault observed is 5,4 seconds. It is considered very short so that DR capacity can be utilized maximum.

- Transformer 1 (T1) is connected to Feeder 1 and 4 (F1 and F4) through MV busbar 1. Transformer 2 (T2) is connected to Feeder 2 and 3 (F2 and F3) through MV busbar 2. Transformer 3 (T3) is connected to Feeder 5 and 8 (F5 and F8) through MV busbar 3. Transformer 4 (T4) is connected to Feeder 6 and 7 (F6 and F7) through MV busbar 4.
- Load at feeders F1 and F2, F7 and F8 are of low priority. In case of capacity limitation these will be disconnected.
- IEEE std.1159-2009 defines an interruption as an event during which the voltage is lower than 0,1 p.u.[17]. A sustained interruption is defined as an interruption with duration longer than 1 minute in IEEE std.1366-2003 [18].Here in mathematical analysis only when a load is disconnected from the system, it is called an interruption. [2]
- DR programs are fully functional for cases where DR is considered.
- Power factor of network is 1, which means reactive power is neglected.

5.4 Case Studies

The aim of the case studies is to show the decrease in outage cost due to DR for increased load without investing in the capacity of the network. Load side indices will also be found. First, outage cost will be calculated for increased load without increasing capacity of any component. Then outage cost will be found incorporating different values of DR. The decrease in outage cost is the benefit of DR. Similarly effect of EVs will be evaluated.

5.4.1 Base Cases: Case 1

Considering loading and other parameters as mentioned in the previous section, the outage cost at present of network is found. At present there is no outage due to capacity; outage cost is only because certain time span is required to connect the reserve supply in case of faults in distribution network.

If components capacity is proportionally increased along with increase in load each year, the interruption frequency and duration remains same. However, outage cost will increase due to increase in load. In case reserve capacity of components is not increased with load growth then difference in outage cost depends on load growth.

For different load growth outage cost per year with and without reserve capacity increase is shown in Table 5-4.

Table 5-4: Outage cost for base cases.

Base Case	Load Growth (%)	Outage Cost with Capacity Increase (€/a)	Outage Cost without Capacity Increase (€/a)	Increase in Outage Cost (€/a)
1	0	4 057,49	4 057,49	-
1a	5	4 260,36	87 643,05	83 382,68
1b	10	4 463,24	372 946,10	368 482,86
1c	15	4 666,11	827 756,15	823 090,04
1d	20	4 868,99	1 591 934,35	1 587 065,36
1e	25	5 071,86	2 626 270,67	2 621 198,80
1f	30	5 274,74	3 766 802,25	3 761 527,51
1g	40	5 680,49	6 480 238,22	6 474 557,73
1h	50	6 086,24	9 486 861,76	9 480 775,53

Increase in outage cost is due to following reasons.

1. During fault in sub-transmission network, load at HV/MV substation is higher than capacity of network. Load is required to be curtailed for duration of HV line repair.

2. For fault on HV busbar 7, loads are required to transfer to neighboring HV/MV substation. During this fault, after removing faulty section of busbar with help of tie, only one transformer remains energized.

3. For transformer and MV busbar faults loads are required to transfer to neighboring HV/MV substation as reserve in same substation can only take load partially.

4. For MV feeder faults on F1 and F2 near HV/MV substation, load curtailment is required to avoid overload of feeder.

For higher load growth more load is required to be curtailed or transferred to neighboring substation. In Figure 5-6 through 5-9 it can be observed that interruption frequency and duration increases with load growth for low priority loads (loads on F1, F2, F7 and F8).

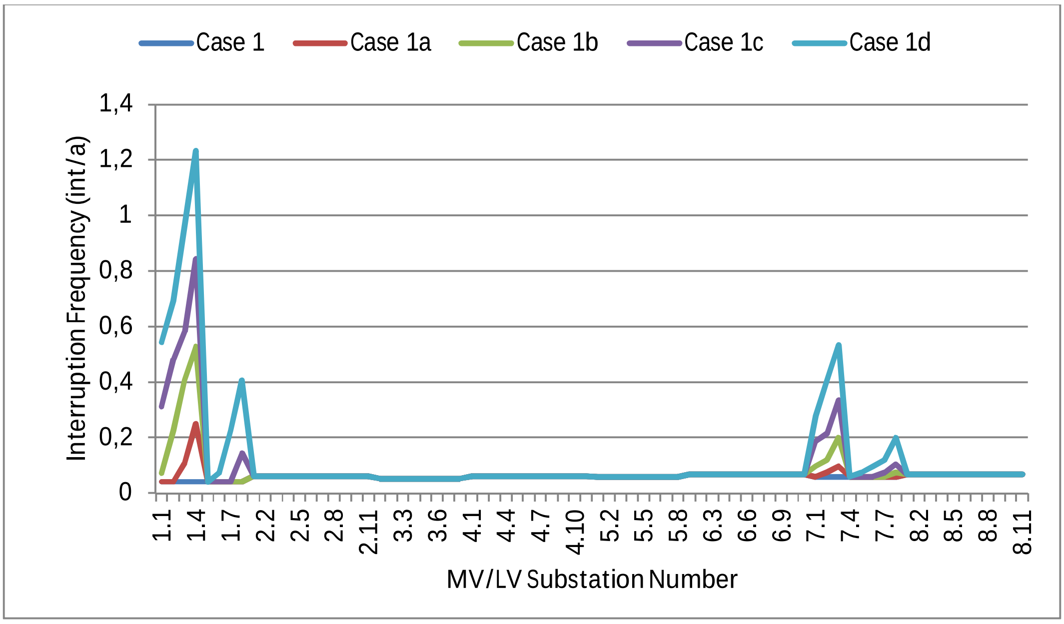

Figure 5-6: Annual interruption frequency for MV/LV substations (Base cases 1-1d)

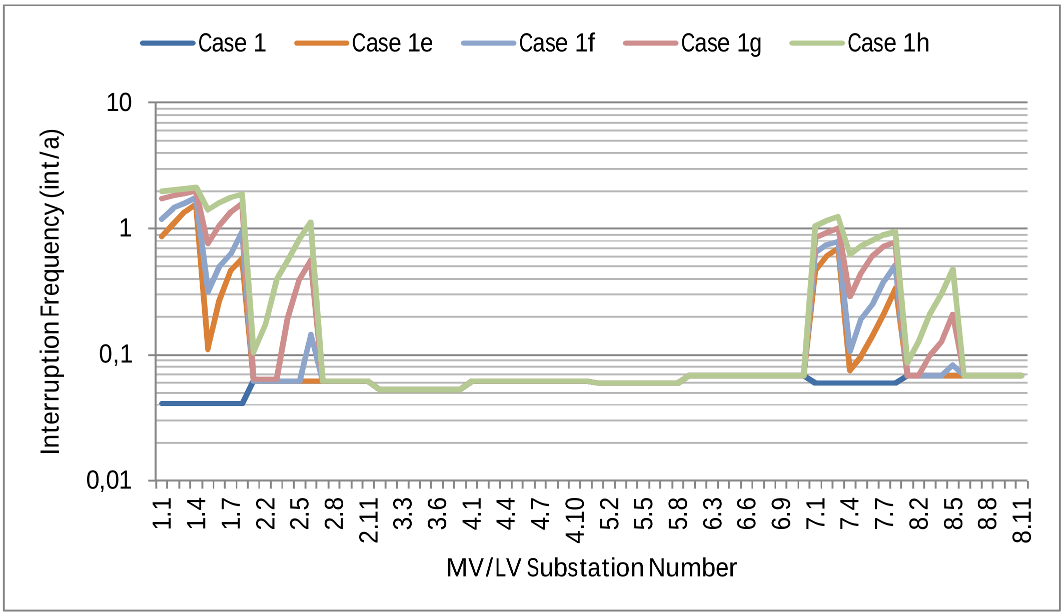

Figure 5-7: Annual interruption frequency for MV/LV substations (Base cases 1e-1h)

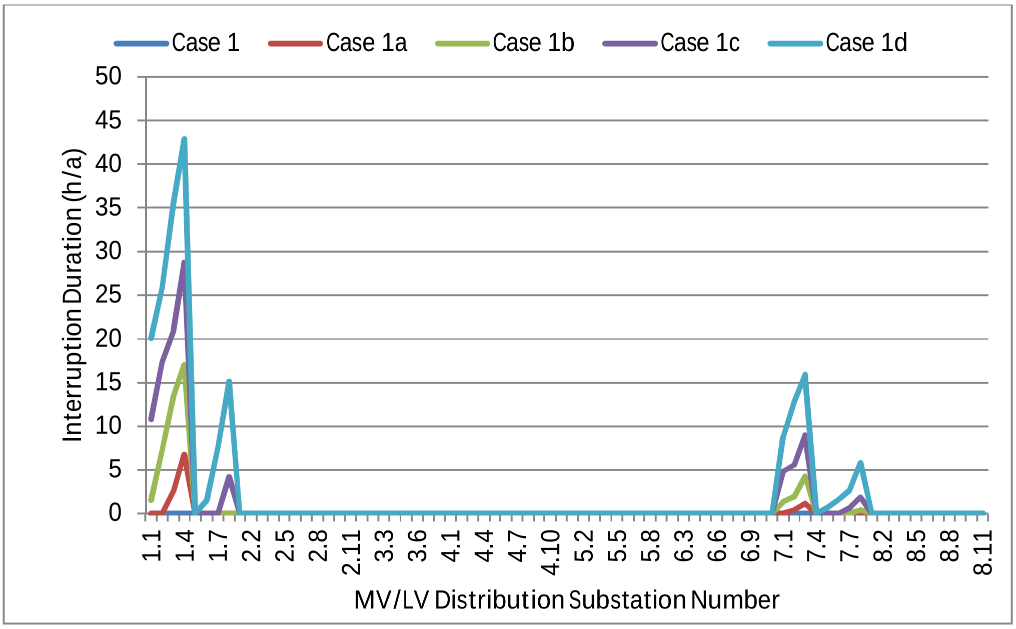

Figure 5-8: Annual outage duration for MV/LV substations (Base cases 1-1d)

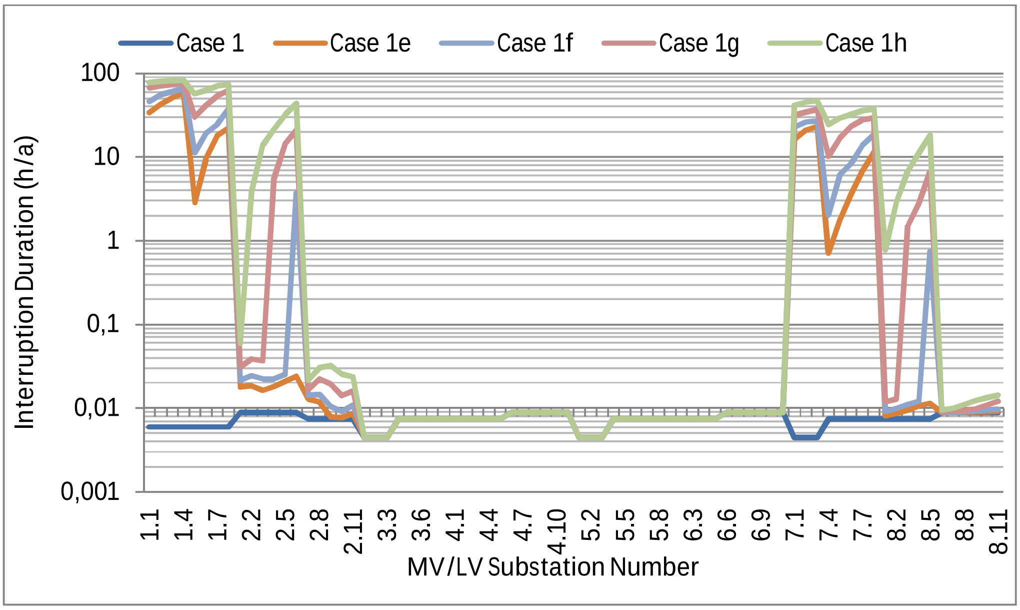

Figure 5-9: Annual outage duration for MV/LV substation (Base cases 1e-1h)

5.4.2 Demand Response (DR) Cases

Outage cost increase calculated in previous section is very high. These high outage costs are not acceptable. For following scenarios of DR capacity (C_{DR}) and demand postponement time (T_{DR}) the decrease in outage cost is calculated.

a) C_{DR} = 20% and T_{DR} = 1h

b) C_{DR} = 20% and T_{DR} = 2h

c) C_{DR} = 20% and T_{DR} = 5h

d) C_{DR} = 35% and T_{DR} = 1h

e) C_{DR} = 35% and T_{DR} = 2h

f) C_{DR} = 35% and T_{DR} = 5h

g) C_{DR} = 50% and T_{DR} = 1h

h) C_{DR} = 50% and T_{DR} = 2h

i) C_{DR} = 50% and T_{DR} = 5h

.

Case 2: Load Growth 5% with DR

The interruption frequency remains same as base case 1a; however, interruption duration of loads changes depending on the capacity and postponement time of DR. With the increase in either DR capacity or load postponement time, interruption duration of load will decrease. Consequently outage cost will be decreased. Decrease in outage cost is shown in Table 5-5 and Figure 5-10. Corresponding interruption durations of distribution substations are indicated in Figures 5-11 to 5-13

Table 5-5: Decrease in Outage Cost due to DR (Case 2)

Case 2	DR Capacity (%)	DR Postponement Time (h)	Outage Cost with capacity Increase (€/a)	Outage Cost without Capacity Increase (€/a)	Outage Cost with DR (€/a)	Decrease in Outage Cost (%)
a	20	1	4260,36	87 643,05	72933,33	17,64
b	20	2	4260,36	87 643,05	59822,19	33,37
c	20	5	4260,36	87 643,05	14817,04	87,34
d	35	1	4260,36	87 643,05	60134,30	32,99
e	35	2	4260,36	87 643,05	23923,09	76,42
f	35	5	4260,36	87 643,05	4260,36	100,00
g	50	1	4260,36	87 643,05	43683,27	52,72
h	50	2	4260,36	87 643,05	14817,04	87,34
i	50	5	4260,36	87 643,05	4260,36	100,00

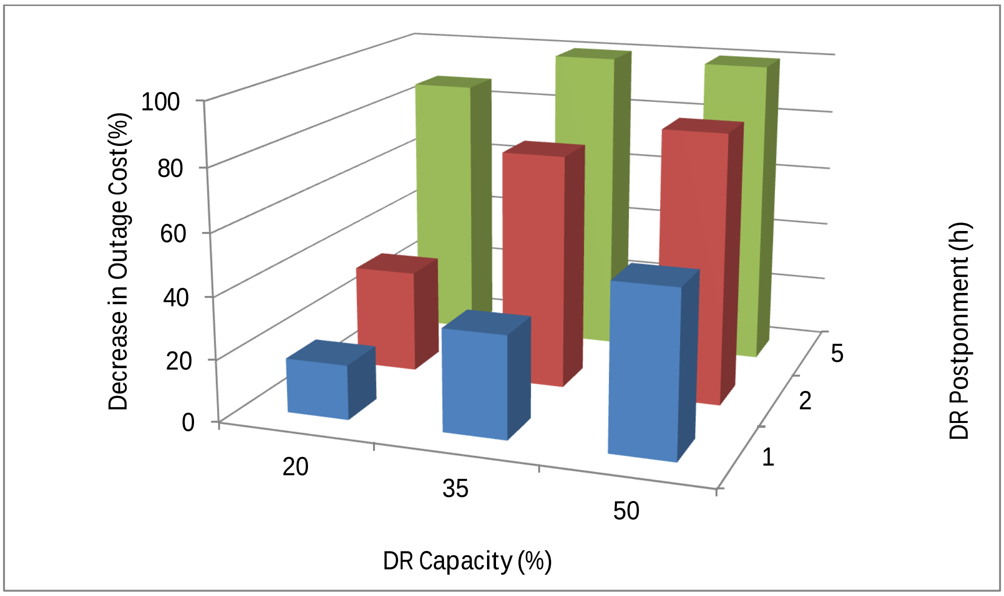

Figure 5-10: Decrease in Outage Cost due to DR (Case 2)

For cases 2a, 2b and 2d decrease in load due to DR is small, all types of faults results in capacity constraint and loads are required to be curtailed or transferred. Thus no considerable decrease in outage cost is gained.

For case 2g there is improvement in decrease in outage cost as DR is able to compensate capacity constraint for cables faults. Cables have shortest repair time. All other types of faults results in capacity constraint and loads are required to be curtailed or transferred.

For cases 2c, 2e and 2h even more improvement is observed because DR is able to compensate capacity constraint for cables and MV busbars faults. MV busbars and cables repair times are shorter and lesser than 24 hours. However, still DR is not able to compensate fully for HV network and HV/MV transformer faults as repair time of these are higher than 24 hours. Due to repair time higher than 24 hours, entire DR resource cannot be used at same time; DR resources are required to be used sequentially in form of small groups.

For cases 2f and 2i decrease in outage cost is 100% because DR is able to fully compensate capacity constraint for all types of faults.

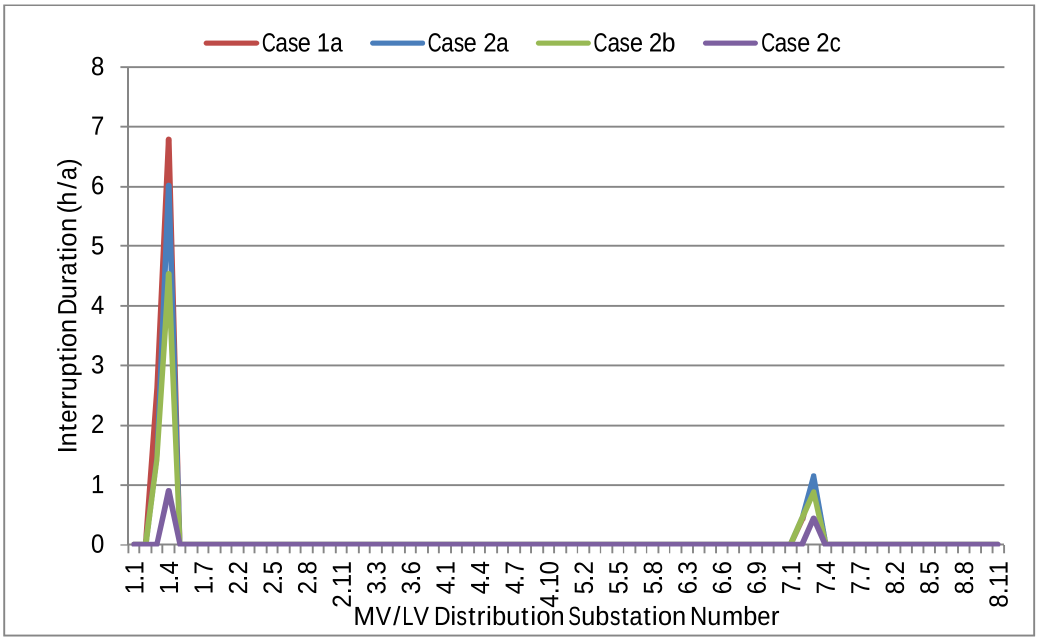

Figure 5-11: Annual interruption duration for MV/LV substations (Case 2a-2c)

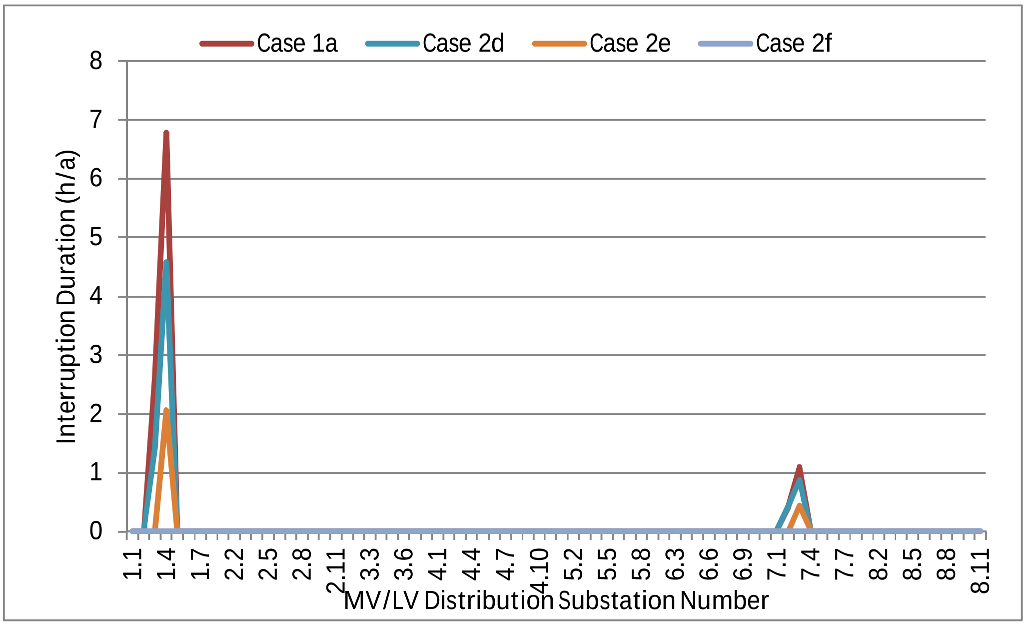

Figure 5-12: Annual interruption duration for MV/LV substations (Case 2d-2f)

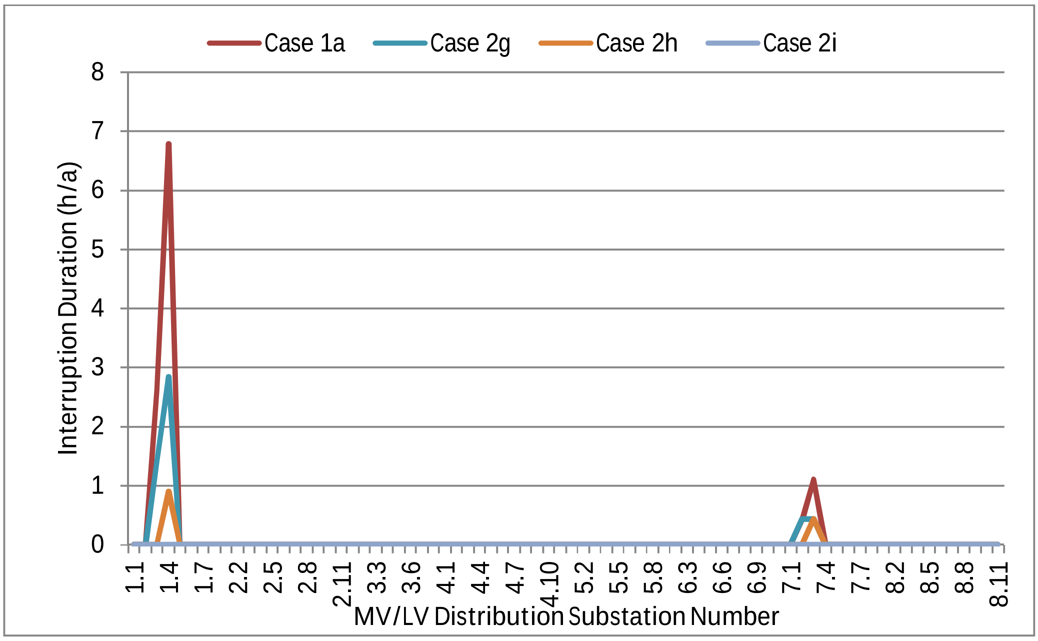

Figure 5-13: Annual interruption duration for MV/LV substations (Case 2g-2i)

Case 3: Load Growth 10% with DR

Decrease in outage cost and corresponding interruption durations of distribution substations are shown in Table 5-6 and Figures 5-14 to 5-17. Higher decrease in outage cost with increase in capacity and postponement time of DR is due to reduced requirement of load curtailment or transfer (similar to case 2). However overall decrease in outage cost is reduced due to higher load growth.

Table 5-6: Decrease in Outage Cost due to DR (Case 3)

Case 3	DR Capacity (%)	DR Postponement Time (h)	Outage Cost with capacity Increase (€/a)	Outage Cost without Capacity Increase (€/a)	Outage Cost with DR (€/a)	Decrease in Outage Cost (%)
a	20	1	4 463,24	372 946,10	311 068,25	16,79
b	20	2	4 463,24	372 946,10	206 422,12	45,19
c	20	5	4 463,24	372 946,10	87 820,73	77,38
d	35	1	4 463,24	372 946,10	233 663,40	37,80
e	35	2	4 463,24	372 946,10	150 916,24	60,26
f	35	5	4 463,24	372 946,10	25 164,91	94,38
g	50	1	4 463,24	372 946,10	199 097,93	47,18
h	50	2	4 463,24	372 946,10	87 820,73	77,38
i	50	5	4 463,24	372 946,10	4 463,24	100,00

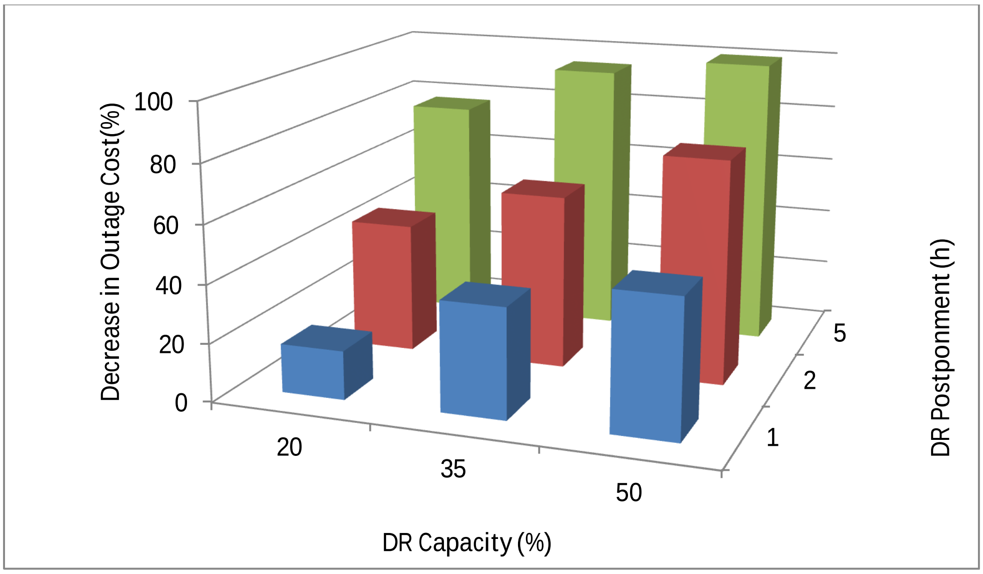

Figure 5-14: Decrease in outage cost due to DR (Case 3)

For cases 3a, 3b, 3d, 3e and 3g decrease in load due to DR is small, all types of faults results in capacity constraint and loads are required to be curtailed or transferred. Thus decrease in outage cost is not worthwhile.

For cases 3c and 3h, there is improvement in decrease in outage cost as DR is able to compensate capacity constraint for cables faults. Cables have shortest repair time.

All other types of faults results in capacity constraint and loads are required to be curtailed or transferred.

For case 3f even more improvement is observed because DR is able to compensate capacity constraint for cables and MV busbars faults. MV busbars and cables repair times are shorter and lesser than 24 hours. However, still DR is not able to compensate fully for HV network and HV/MV transformer faults as repair time of these are higher than 24 hours. Due to repair time higher than 24 hours, entire DR resource cannot be used at same time; DR resources are required to be used sequentially in form of small groups.

For case 3i decrease in outage cost is 100% because DR is able to fully compensate capacity constraint for all types of faults.

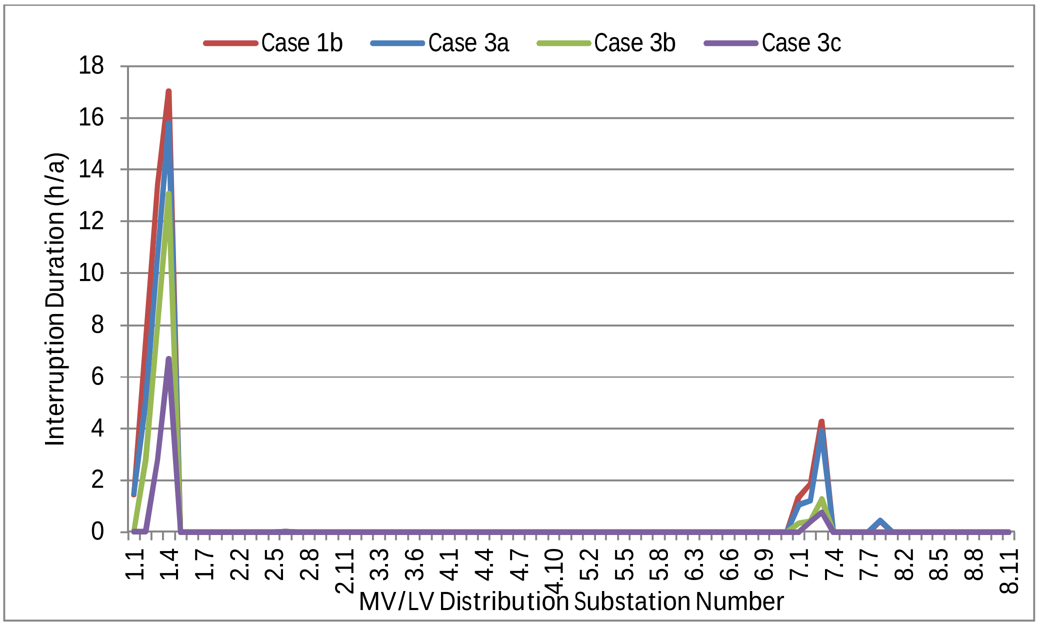

Figure 5-15: Annual interruption duration for MV/LV substations (Case 3a-3c)

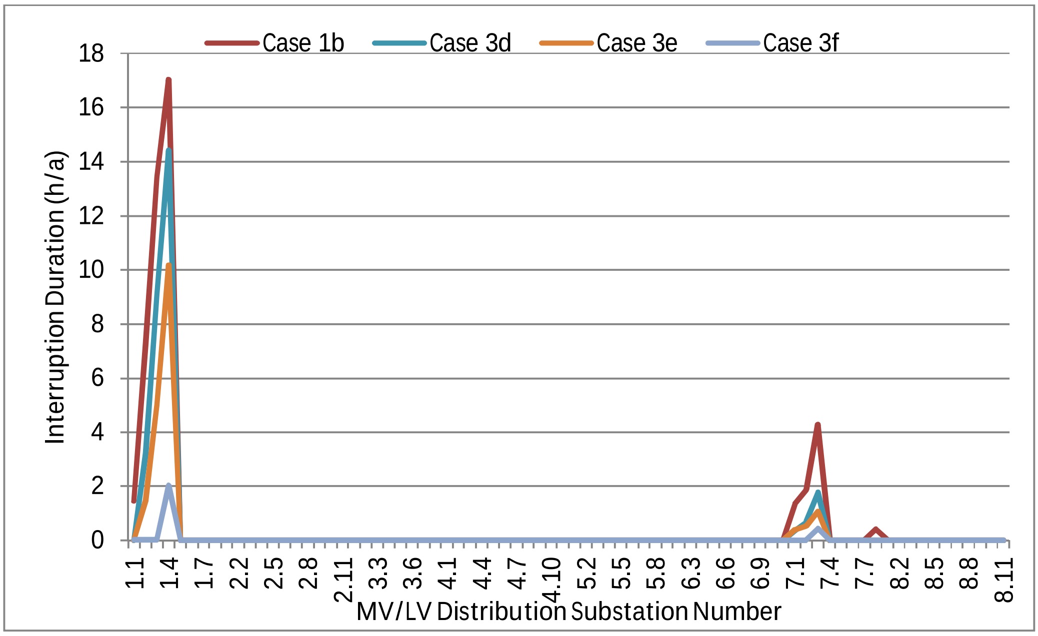

Figure 5-16: Annual interruption duration for MV/LV substations (Case 3d-3f)

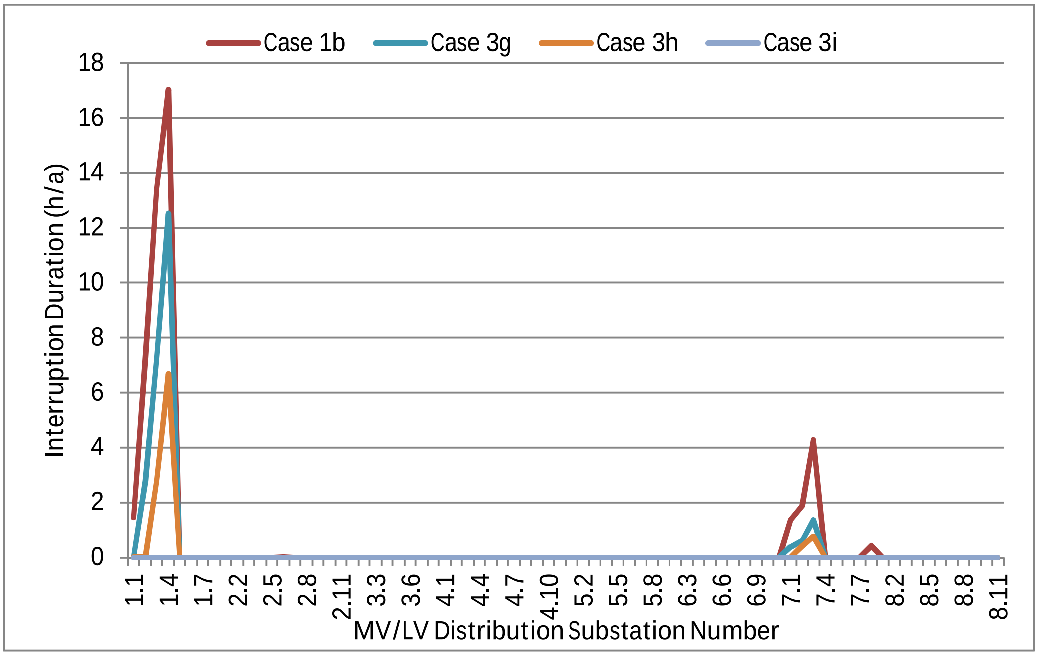

Figure 5-17: Annual interruption duration for MV/LV substations (Case 3g-3i)

Case 4: Load Growth 15% with DR

Decrease in outage cost and corresponding interruption durations of distribution substations are shown in Table 5-7 and Figures 5-18 to 5-21. Higher decrease in outage cost with increase in capacity and postponement time of DR is due to reduced requirement of load curtailment or transfer (similar to case 2). However overall decrease in outage cost is reduced due to higher load growth.

Table 5-7: Decrease in Outage Cost due to DR (Case 4)

Case 4	DR Capacity (%)	DR Postponement Time (h)	Outage Cost with capacity Increase (€/a)	Outage Cost without Capacity Increase (€/a)	Outage Cost with DR (€/a)	Decrease in Outage Cost (%)
a	20	1	4 666,11	827 756,15	639 541,20	22,87
b	20	2	4 666,11	827 756,15	551 725,09	33,54
c	20	5	4 666,11	827 756,15	298 858,52	64,26
d	35	1	4 666,11	827 756,15	556 842,11	32,91
e	35	2	4 666,11	827 756,15	353 704,20	57,59
f	35	5	4 666,11	827 756,15	152 063,00	82,09
g	50	1	4 666,11	827 756,15	497 530,49	40,12
h	50	2	4 666,11	827 756,15	298 858,52	64,26
i	50	5	4 666,11	827 756,15	52 953,58	94,13

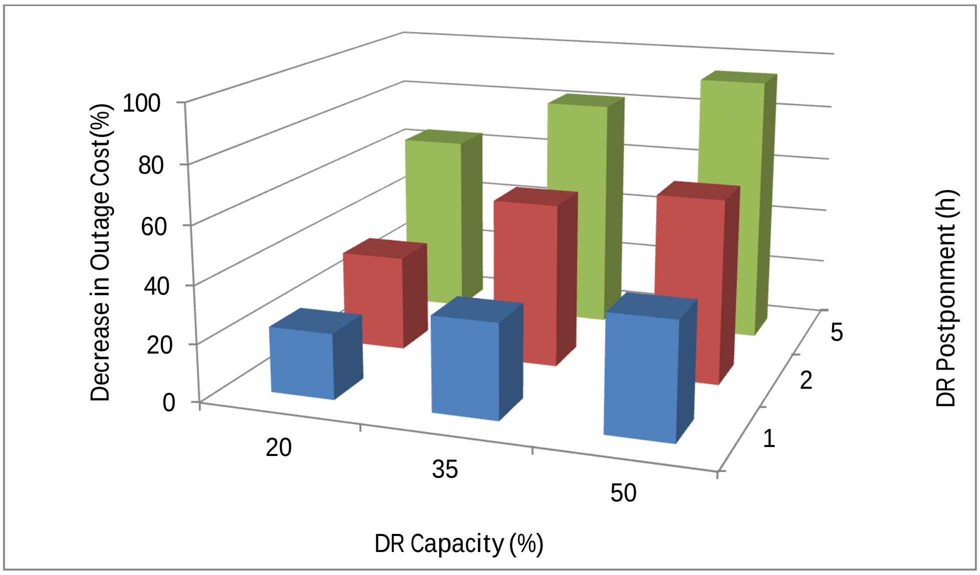

Figure 5-18: Decrease in outage cost due to DR (Case 4)

For cases 4a, 4b, 4c, 4d, 4e, 4g and 4h decrease in load due to DR is small, all types of faults results in capacity constraint and loads are required to be curtailed or transferred. Thus decrease in outage cost is not worthwhile.

For case 4f, there is improvement in decrease in outage cost as DR is able to compensate capacity constraint for cables faults. Cables have shortest repair time. All other types of faults results in capacity constraint and loads are required to be curtailed or transferred.

For case 4i even more improvement is observed because DR is able to compensate capacity constraint for cables and MV busbars faults. MV busbars and cables repair times are shorter and lesser than 24 hours. However, still DR is not able to compensate fully for HV network and HV/MV transformer faults as repair time of these are higher than 24 hours. Due to repair time higher than 24 hours, entire DR resource cannot be used at same time; DR resources are required to be used sequentially in form of small groups.

For 15% load growth none of considered DR case is able to fully compensate capacity constraint due to high load growth compared to DR load reduction.

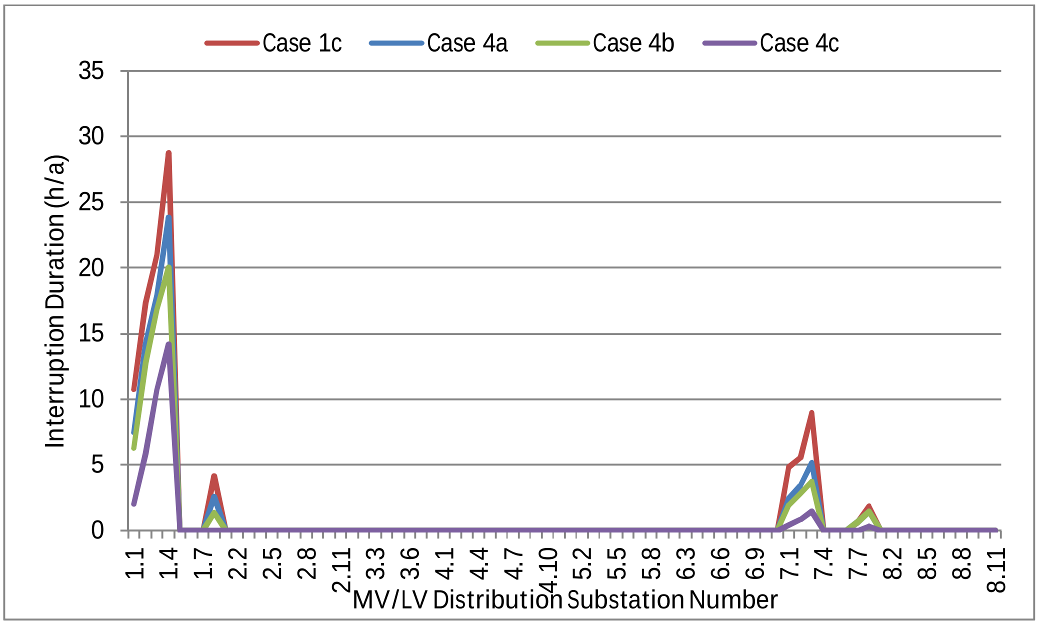

Figure 5-19: Annual interruption duration for MV/LV substations (Case 4a-4c)

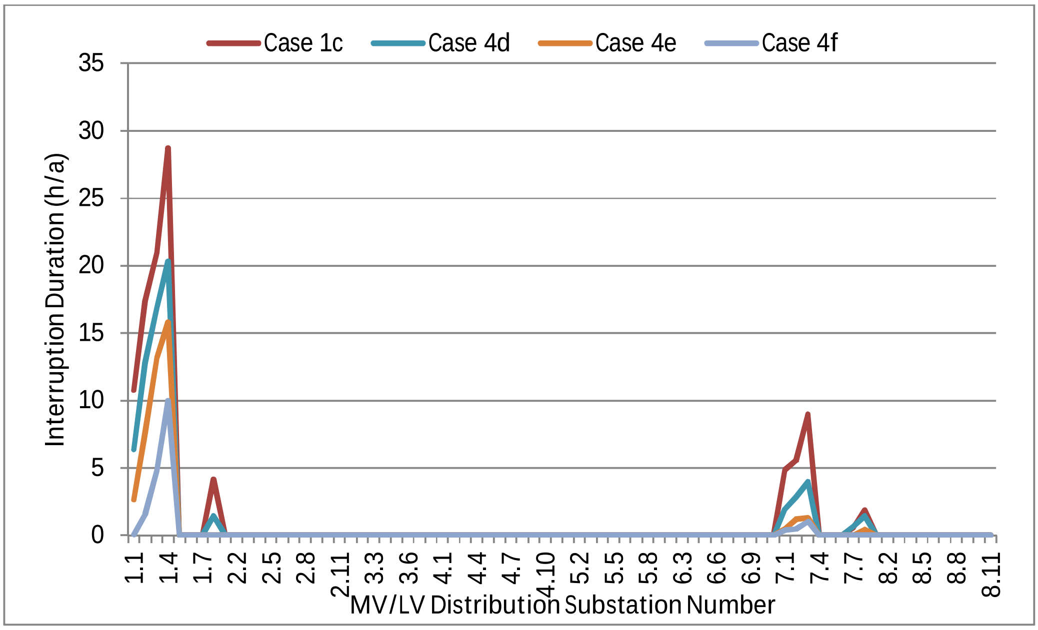

Figure 5-20: Annual interruption duration for MV/LV substations (Case 4d-4f)

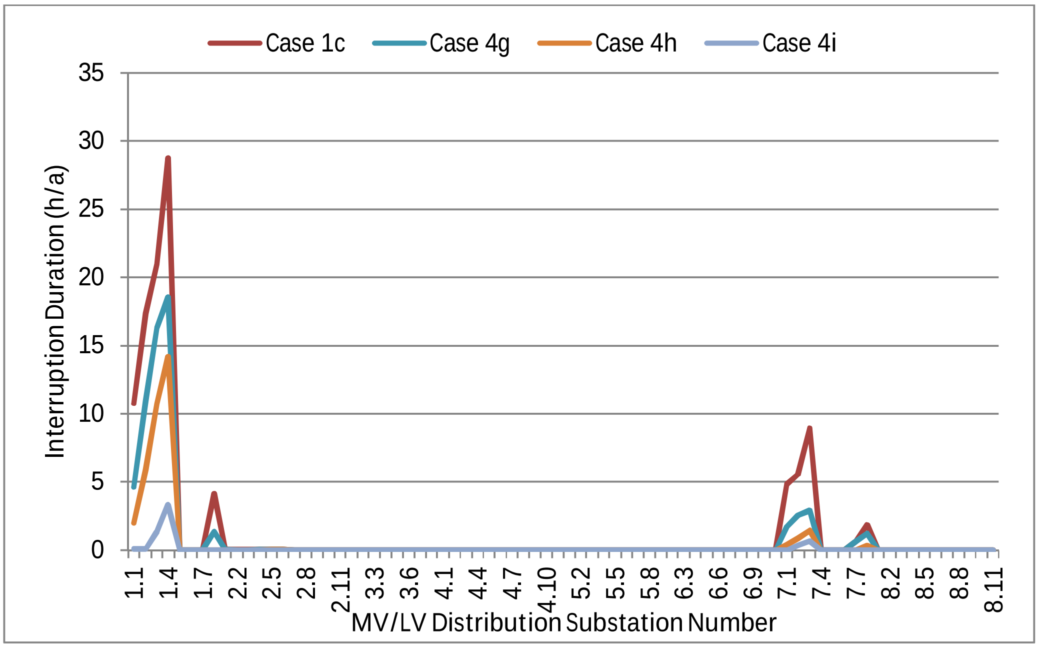

Figure 5-21: Annual interruption duration for MV/LV substations (Case 4g-4i)

Case 5: Load Growth 20% with DR

Decrease in outage cost and corresponding interruption durations of distribution substations are shown in Table 5-8 and Figures 5-22 to 5-25. Higher decrease in outage cost with increase in capacity and postponement time of DR is due to reduced requirement of load curtailment or transfer (similar to case 2). However overall decrease in outage cost is reduced due to higher load growth.

Table 5-8: Decrease in Outage Cost due to DR (Case 5)

Case 5	DR Capacity (%)	DR Postponement Time (h)	Outage Cost with capacity Increase (€/a)	Outage Cost without Capacity Increase (€/a)	Outage Cost with DR (€/a)	Decrease in Outage Cost (%)
a	20	1	4 868,99	1 591 934,35	1 393 327,44	12,51
b	20	2	4 868,99	1 591 934,35	986 565,14	38,14
c	20	5	4 868,99	1 591 934,35	551 120,04	65,58
d	35	1	4 868,99	1 591 934,35	1 084 954,46	31,94
e	35	2	4 868,99	1 591 934,35	662 015,54	58,59
f	35	5	4 868,99	1 591 934,35	330 337,82	79,49
g	50	1	4 868,99	1 591 934,35	843 812,84	47,14
h	50	2	4 868,99	1 591 934,35	551 120,04	65,58
i	50	5	4 868,99	1 591 934,35	187 624,47	88,48

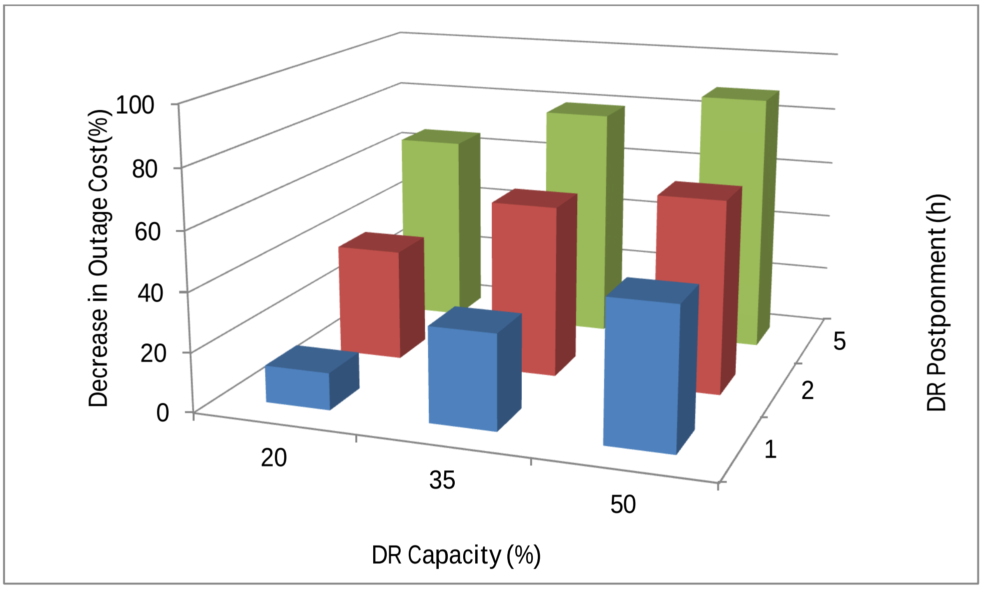

Figure 5-22: Decrease in outage cost due to DR (Case 5)

For cases 5a to 5h decrease in load due to DR is small, all types of faults results in capacity constraint and loads are required to be curtailed or transferred. Thus decrease in outage cost is not worthwhile.

For case 5i improvement is observed because DR is able to compensate capacity constraint for cables and MV busbars faults. MV busbars and cables repair times are shorter and lesser than 24 hours. However, still DR is not able to compensate fully for HV network and HV/MV transformer faults as repair time of these are higher than 24 hours. Due to repair time higher than 24 hours, entire DR resource cannot be used at same time; DR resources are required to be used sequentially in form of small groups.

For 20% load growth none of considered DR cases is able to fully compensate capacity constraint due to high load growth compared to DR load reduction.

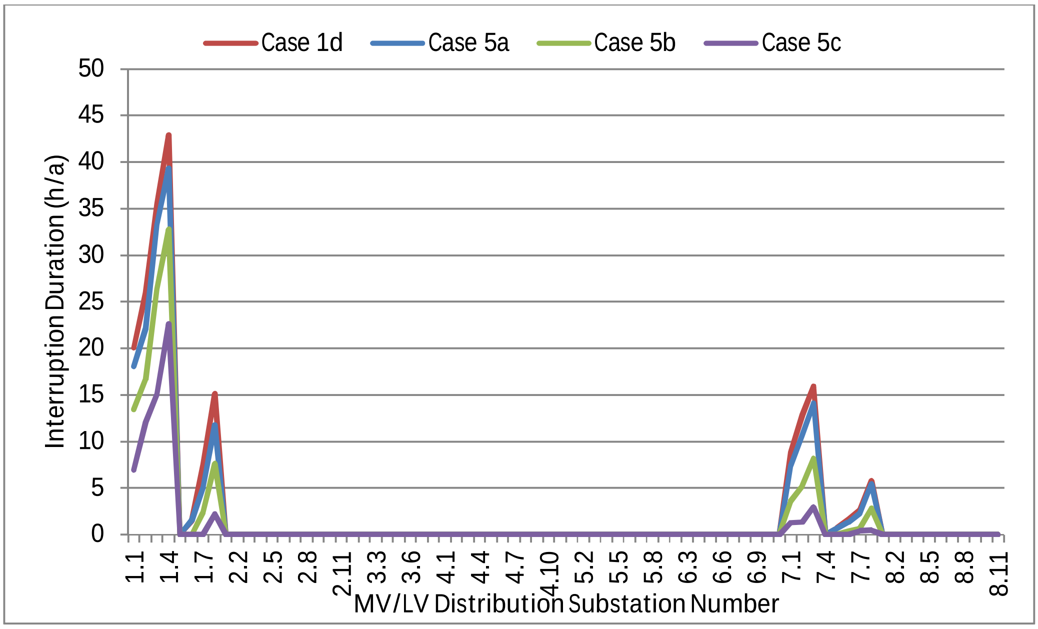

Figure 5-23: Annual interruption duration for MV/LV substations (Case 5a-5c)

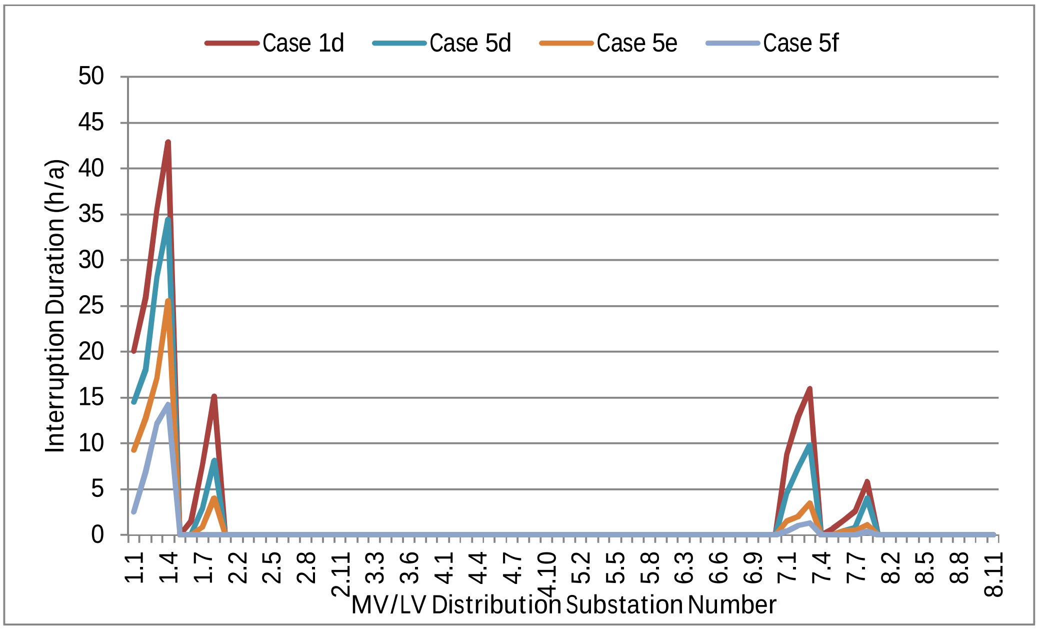

Figure 5-24: Annual interruption duration for MV/LV substations (Case 5d-5f)

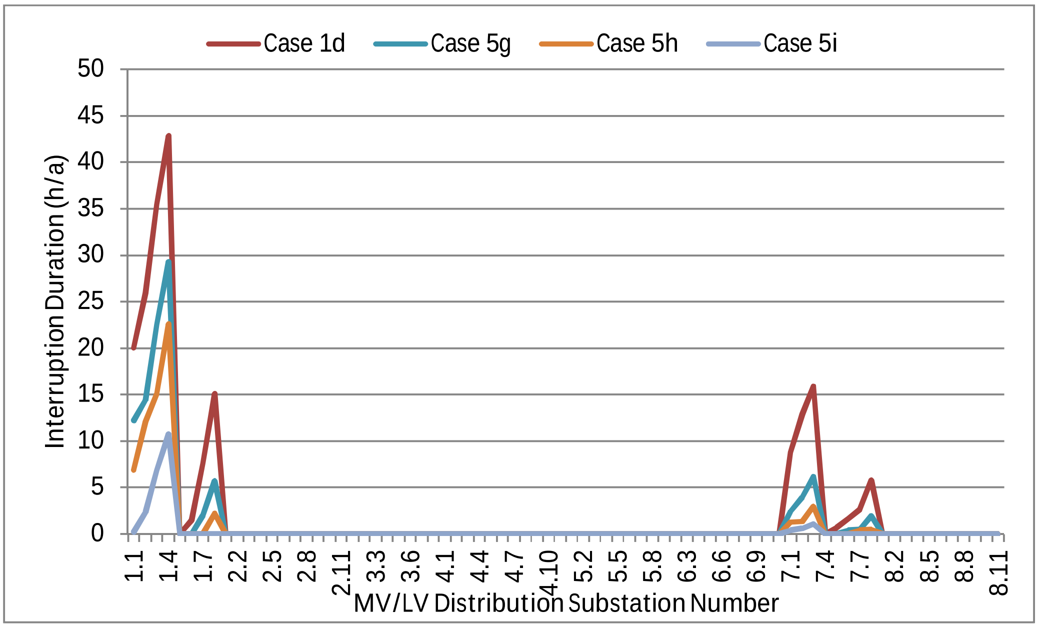

Figure 5-25: Annual interruption duration for MV/LV substations (Case 5g-5i)

Case 6: Load Growth 25% with DR

Decrease in outage cost and corresponding interruption durations of distribution substations are shown in Table 5-9 and Figures 5-26 to 5-29. Higher decrease in outage cost with increase in capacity and postponement time of DR is due to reduced requirement of load curtailment or transfer (similar to case 2). However overall decrease in outage cost compared to previous cases is reduced due to higher load growth.

Table 5-9: Decrease in Outage Cost due to DR (Case 6)

Case 6	DR Capacity (%)	DR Postponement Time (h)	Outage Cost with capacity Increase (€/a)	Outage Cost without Capacity Increase (€/a)	Outage Cost with DR (€/a)	Decrease in Outage Cost (%)
a	20	1	5 071,86	2 626 270,67	2 169 853,57	17,41
b	20	2	5 071,86	2 626 270,67	1 819 130,98	30,79
c	20	5	5 071,86	2 626 270,67	937 410,75	64,43
d	35	1	5 071,86	2 626 270,67	1 928 332,23	26,63
e	35	2	5 071,86	2 626 270,67	1 094 672,85	58,43
f	35	5	5 071,86	2 626 270,67	624 353,44	76,37
g	50	1	5 071,86	2 626 270,67	1 611 121,11	38,73
h	50	2	5 071,86	2 626 270,67	937 410,75	64,43
i	50	5	5 071,86	2 626 270,67	379 978,48	85,70

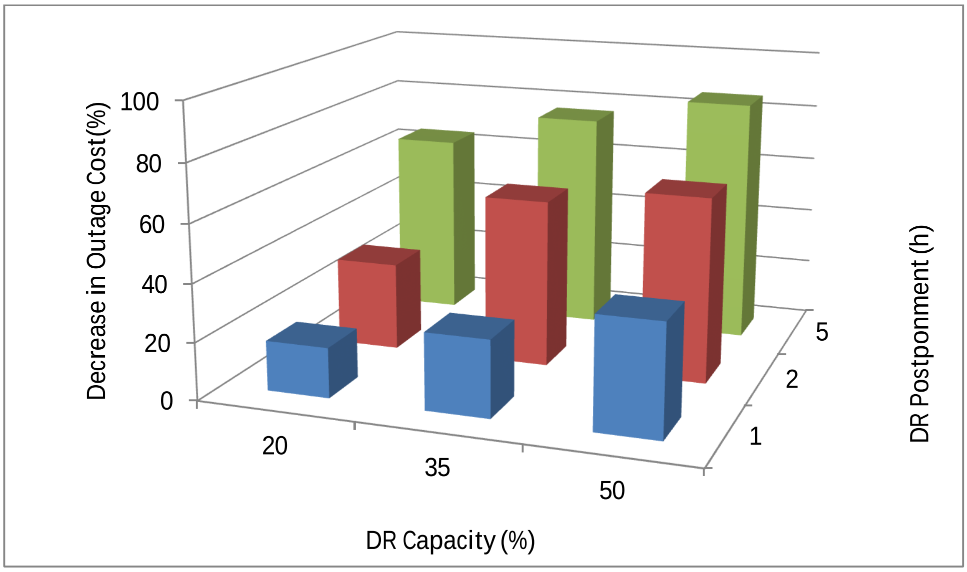

Figure 5-26: Decrease in outage cost due to DR (Case 6)

For cases 6a to 6h decrease in load due to DR is small, all types of faults results in capacity constraint and loads are required to be curtailed or transferred. Thus decrease in outage cost is not worthwhile.

For case 6i, there is improvement in decrease in outage cost as DR is able to compensate capacity constraint for cables faults. Cables have shortest repair time. All other types of faults results in capacity constraint and loads are required to be curtailed or transferred.

For 25% load growth none of considered DR cases is able to fully compensate capacity constraint due to high load growth.

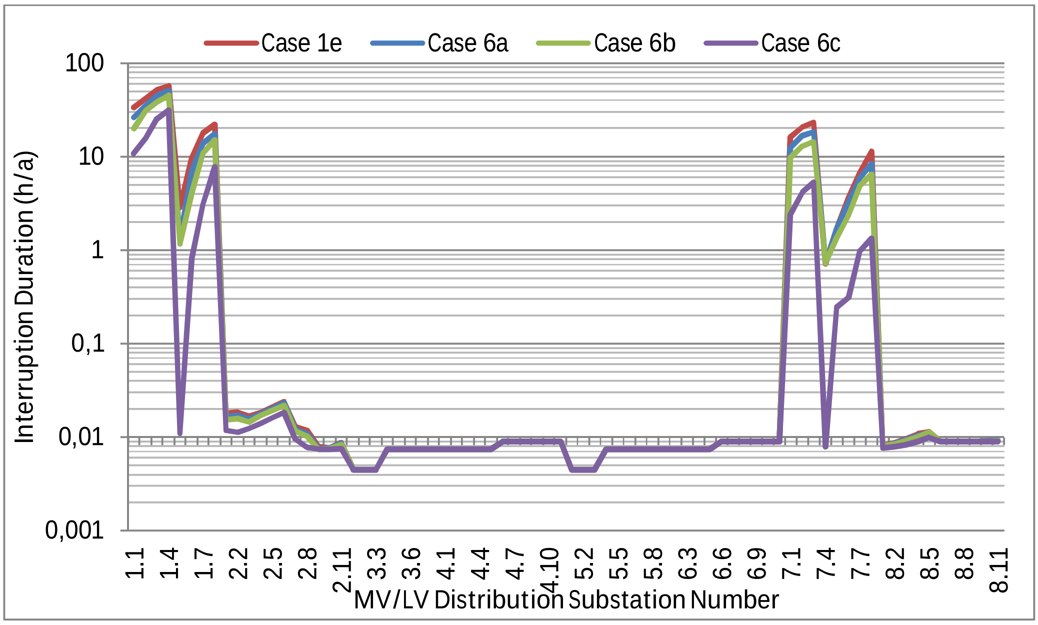

Figure 5-27: Annual interruption duration for MV/LV substations (Case 6a-6c)

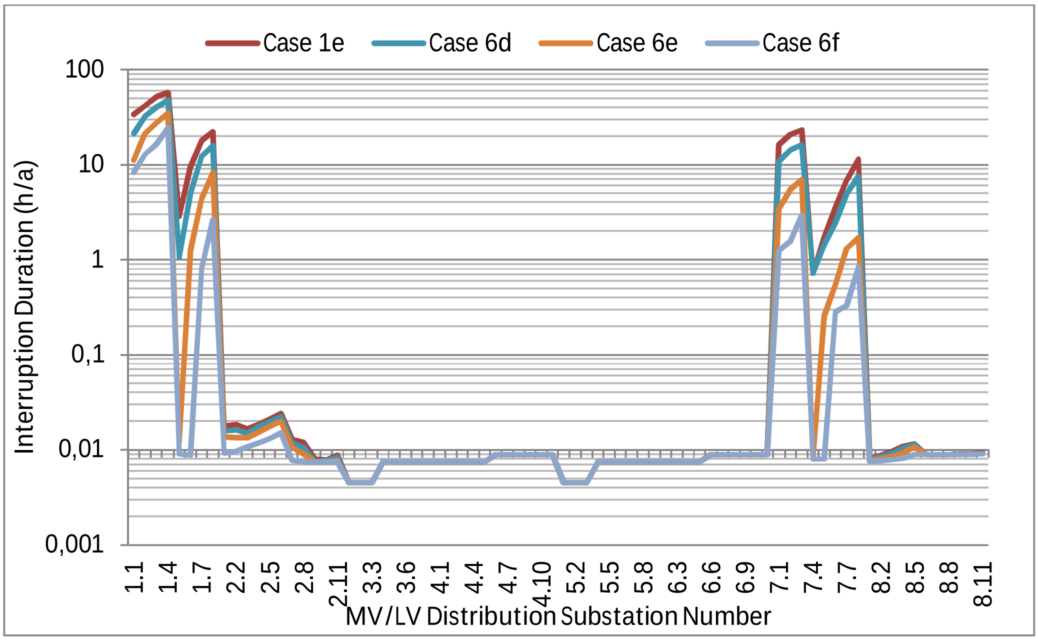

Figure 5-28: Annual interruption duration for MV/LV substations (Case 6d-6f)

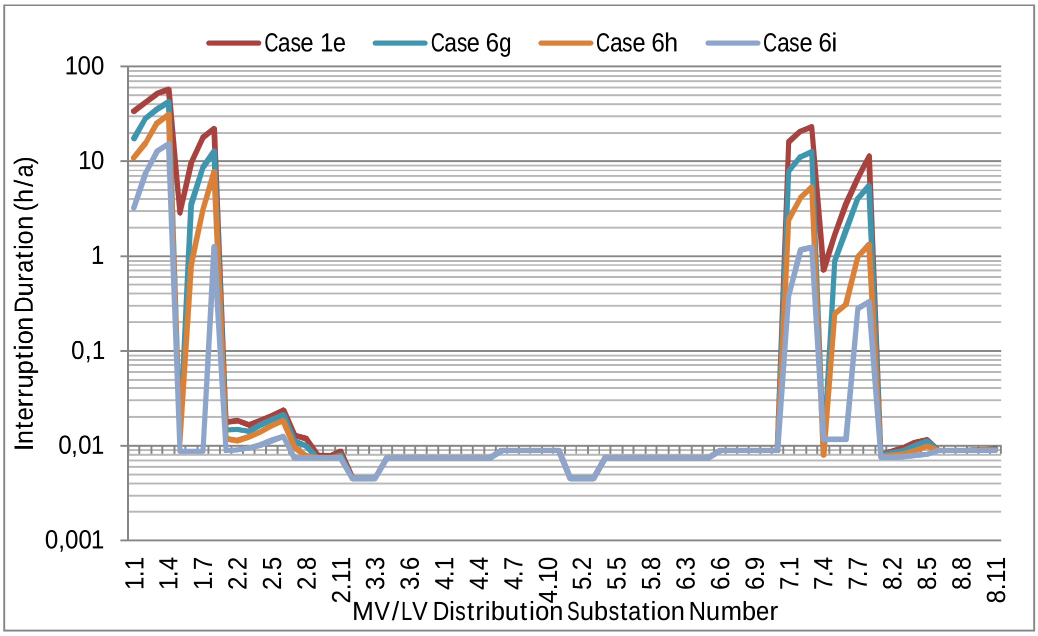

Figure 5-29: Annual interruption duration for MV/LV substations (Case 6g-6i)

Case 7: Load Growth 30% with DR

Decrease in outage cost and corresponding interruption durations of distribution substations are shown in Table 5-10 and Figures 5-30 to 5-33.

Table 5-10: Decrease in Outage Cost due to DR (Case 7)

Case 7	DR Capacity (%)	DR Postponement Time (h)	Outage Cost with capacity Increase (€/a)	Outage Cost without Capacity Increase (€/a)	Outage Cost with DR (€/a)	Decrease in Outage Cost (%)
a	20	1	5 274,74	3 766 802,25	2 926 753,08	22,33
b	20	2	5 274,74	3 766 802,25	2 447 563,06	35,07
c	20	5	5 274,74	3 766 802,25	1 456 444,96	61,42
d	35	1	5 274,74	3 766 802,25	2 585 565,07	31,40
e	35	2	5 274,74	3 766 802,25	1 645 212,27	56,40
f	35	5	5 274,74	3 766 802,25	1 031 719,68	72,71
g	50	1	5 274,74	3 766 802,25	2 147 036,25	43,06
h	50	2	5 274,74	3 766 802,25	1 456 444,96	61,42
i	50	5	5 274,74	3 766 802,25	672 759,19	82,25

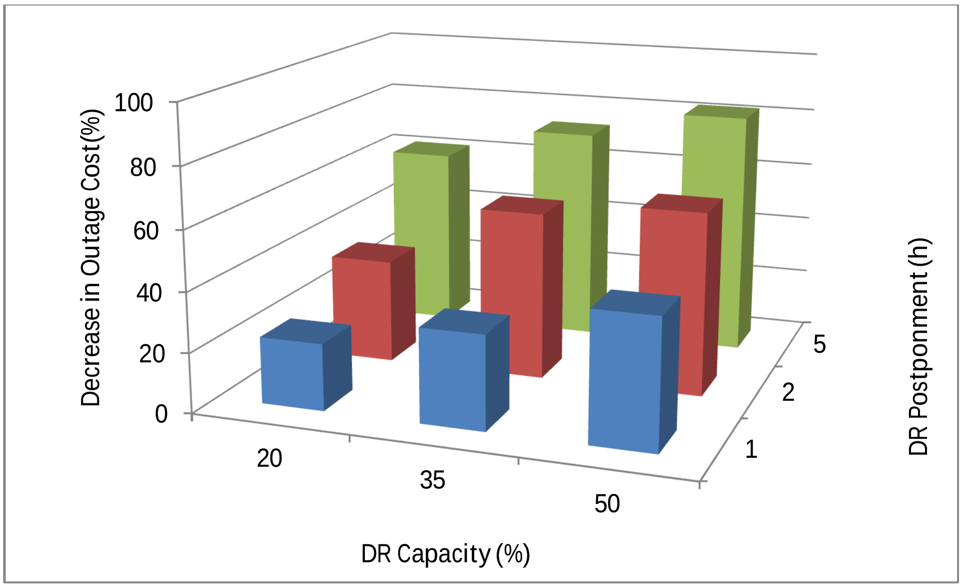

Figure 5-30: Decrease in outage cost due to DR (Case 7)

For cases 7a to 7i decrease in load due to DR is small, all types of faults results in capacity constraint and loads are required to be curtailed or transferred. Higher decrease in outage cost with increase in capacity and postponement time of DR is due to reduced requirement of load curtailment or transfer.

For 30% load growth none of considered DR cases is able to fully compensate capacity constraint of any component, due to high load growth compared to load reduction due to DR.

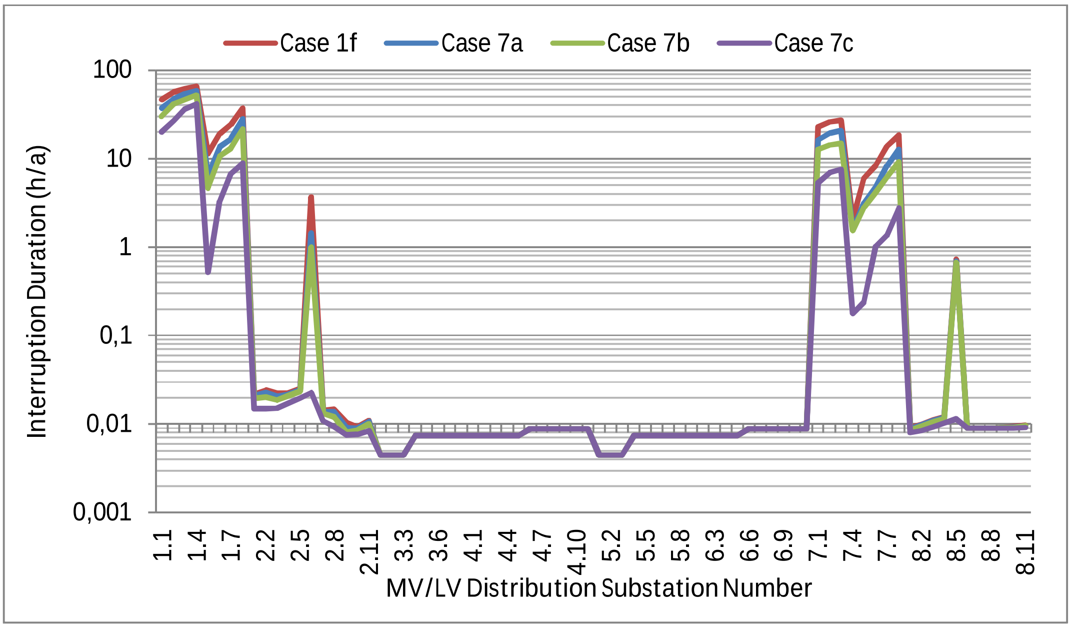

Figure 5-31: Annual interruption duration for MV/LV substations (Case 7a-7c)

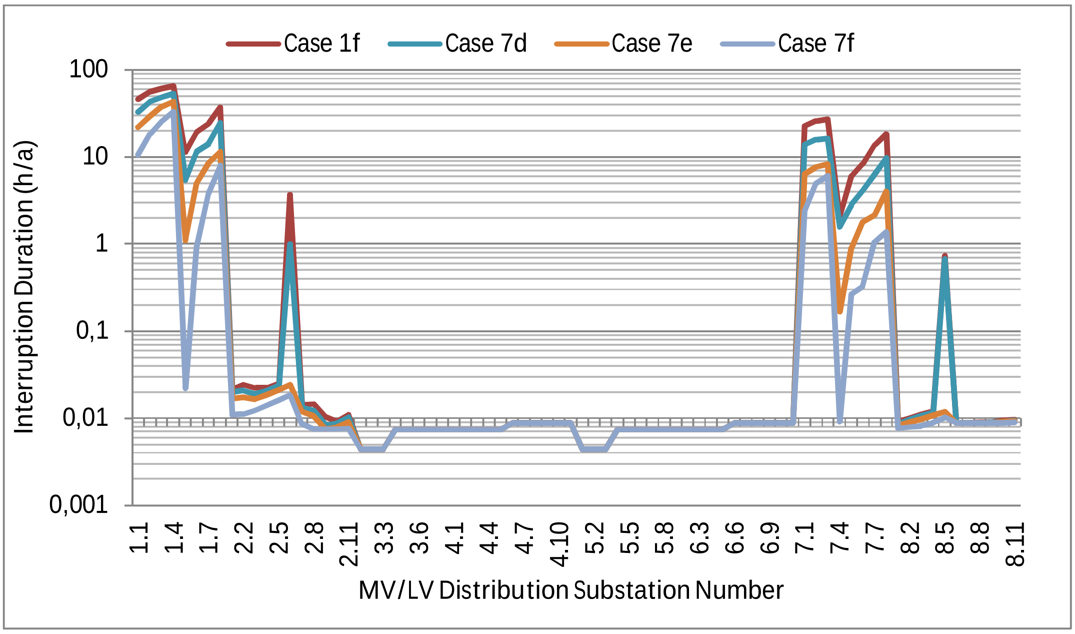

Figure 5-32: Annual interruption duration for MV/LV substations (Case 7d-7f)

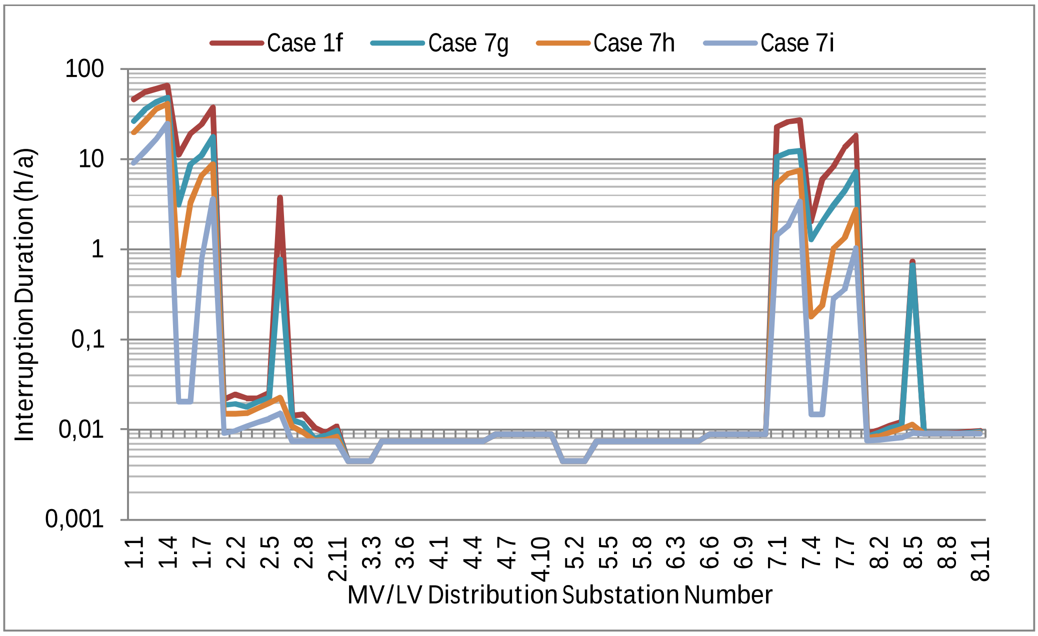

Figure 5-33: Annual interruption duration for MV/LV substations (Case 7g-7i)

Case 8: Load Growth 40% with DR

Decrease in outage cost and corresponding interruption durations of distribution substations are shown in Table 5-11 and Figures 5-34 to 5-37. Similar to case 7 none of considered DR cases is able to fully compensate capacity constraint of any component due to high load growth. Higher decrease in outage cost with increase in capacity and postponement time of DR is due to reduced requirement of load curtailment or transfer.

Table 5-11: Decrease in Outage Cost due to DR (Case 8)

Case 8	DR Capacity (%)	DR Postponement Time (h)	Outage Cost with capacity Increase (€/a)	Outage Cost without Capacity Increase (€/a)	Outage Cost with DR (€/a)	Decrease in Outage Cost (%)
a	20	1	5 680,49	6 480 238,22	5 356 069,84	17,36
b	20	2	5 680,49	6 480 238,22	4 258 869,40	34,31
c	20	5	5 680,49	6 480 238,22	2 527 406,77	61,05
d	35	1	5 680,49	6 480 238,22	4 611 695,00	28,86
e	35	2	5 680,49	6 480 238,22	2 957 094,57	54,42
f	35	5	5 680,49	6 480 238,22	2 050 322,45	68,42
g	50	1	5 680,49	6 480 238,22	3 713 458,16	42,73
h	50	2	5 680,49	6 480 238,22	2 527 406,77	61,05
i	50	5	5 680,49	6 480 238,22	1 560 779,03	75,98

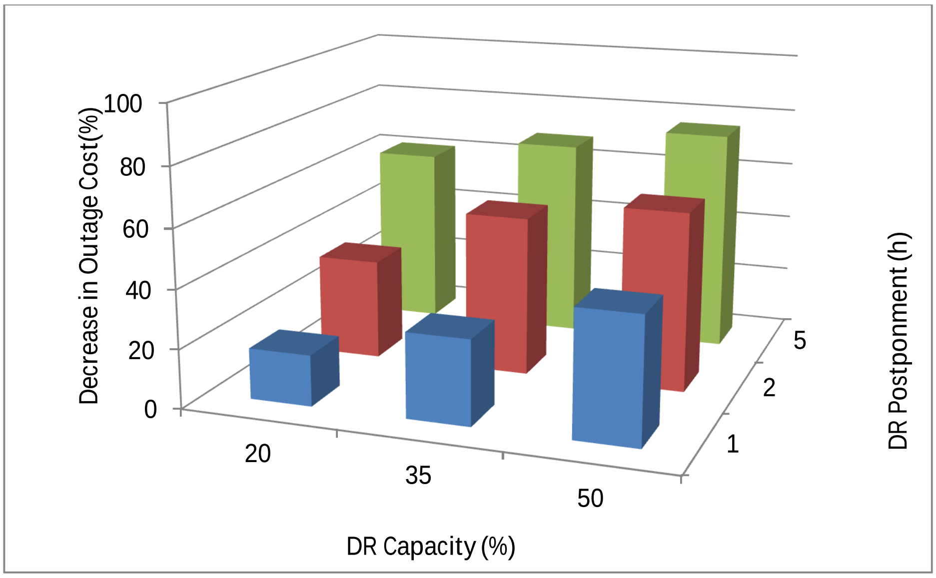

Figure 5-34: Decrease in outage cost due to DR (Case 8)

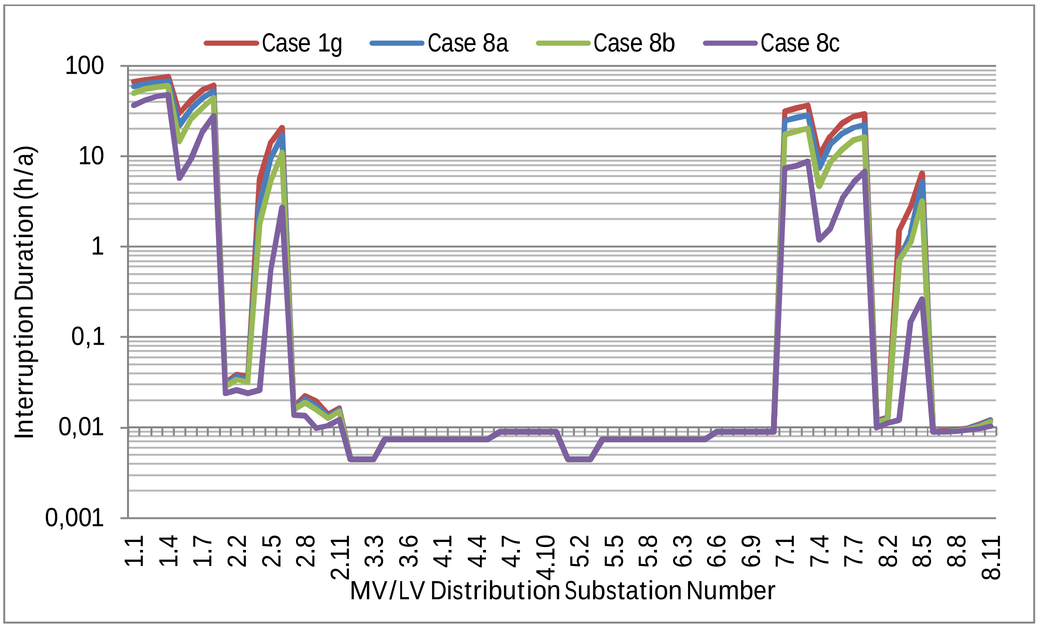

Figure 5-35: Annual interruption duration for MV/LV substations (Case 8a-8c)

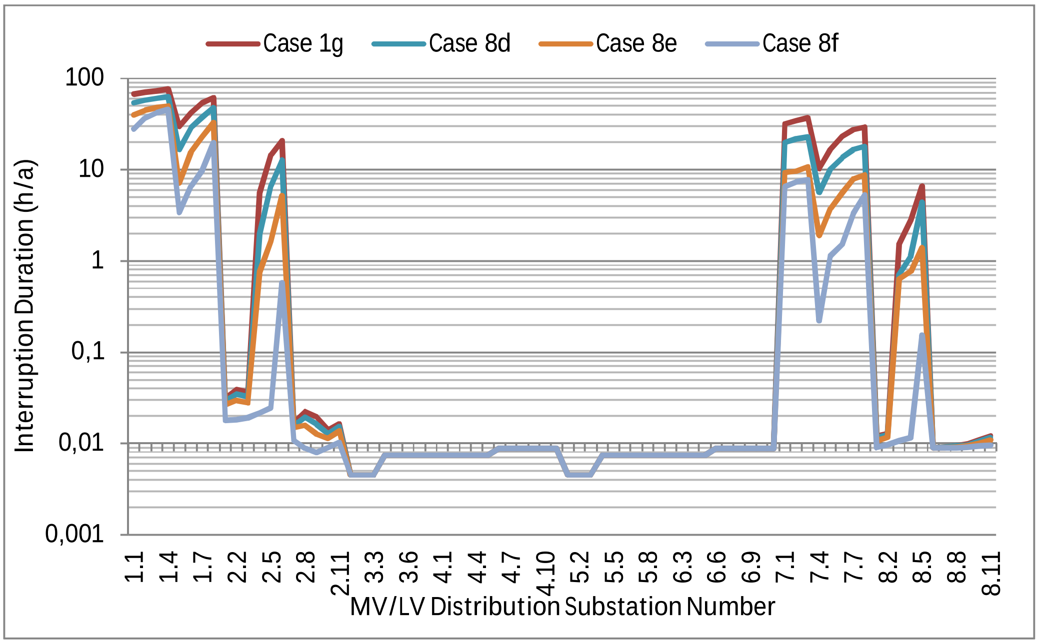

Figure 5-36: Annual interruption duration for MV/LV substations (Case 8d-8f)

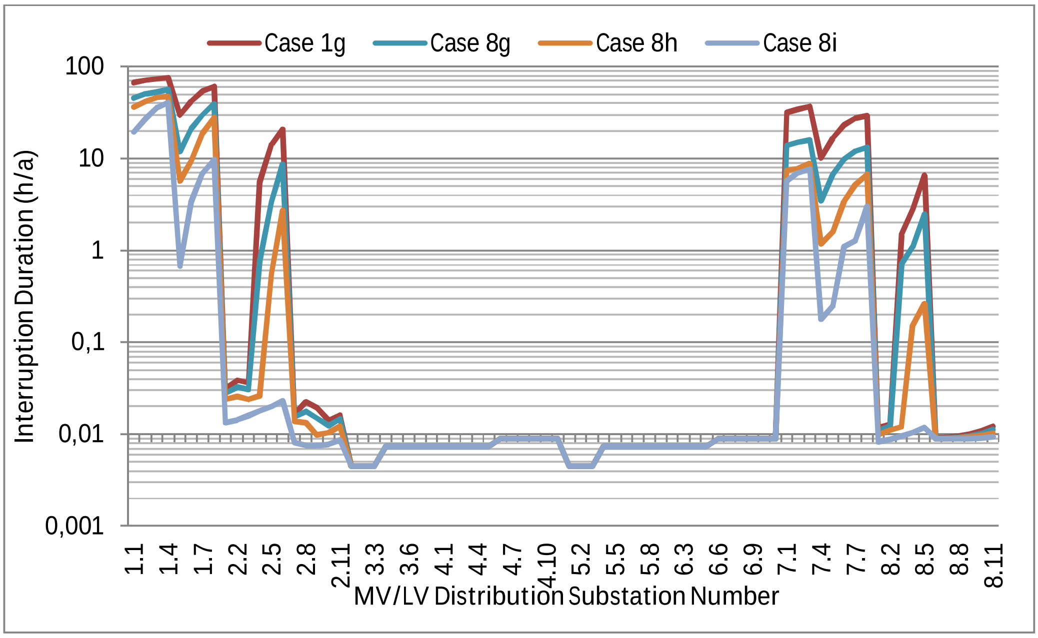

Figure 5-37: Annual interruption duration for MV/LV substations (Case 8g-8i)

Case 9: Load Growth 50% with DR

Decrease in outage cost and corresponding interruption durations of distribution substations are shown in Table 5-12 and Figures 5-38 to 5-41. Similar to case 7 none of considered DR cases is able to fully compensate capacity constraint of any component due to high load growth. Higher decrease in outage cost with increase in capacity and postponement time of DR is due to reduced requirement of load curtailment or transfer.

Table 5-12: Decrease in Outage Cost due to DR (Case 9)

Case 9	DR Capacity (%)	DR Postponement Time (h)	Outage Cost with capacity Increase (€/a)	Outage Cost without Capacity Increase (€/a)	Outage Cost with DR (€/a)	Decrease in Outage Cost (%)
a	20	1	6 086,24	9 486 861,76	8 118 954,71	14,43
b	20	2	6 086,24	9 486 861,76	6 490 421,64	31,61
c	20	5	6 086,24	9 486 861,76	3 580 503,10	62,30
d	35	1	6 086,24	9 486 861,76	7 078 926,46	25,40
e	35	2	6 086,24	9 486 861,76	4 271 345,94	55,01
f	35	5	6 086,24	9 486 861,76	3 171 515,50	66,61
g	50	1	6 086,24	9 486 861,76	5 751 051,90	39,40
h	50	2	6 086,24	9 486 861,76	3 580 503,10	62,30
i	50	5	6 086,24	9 486 861,76	2 672 768,89	71,87

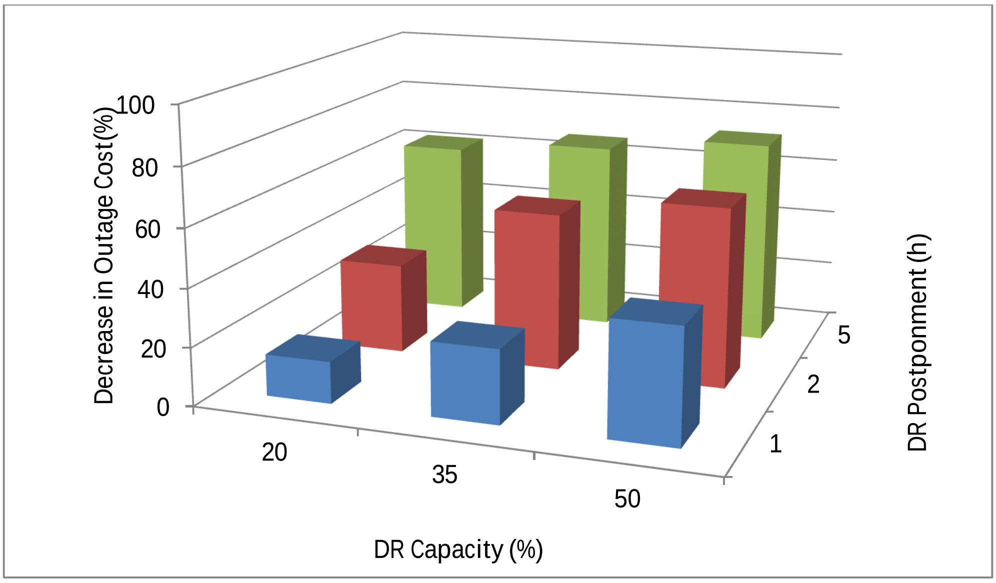

Figure 5-38: Decrease in outage cost due to DR (Case 9)

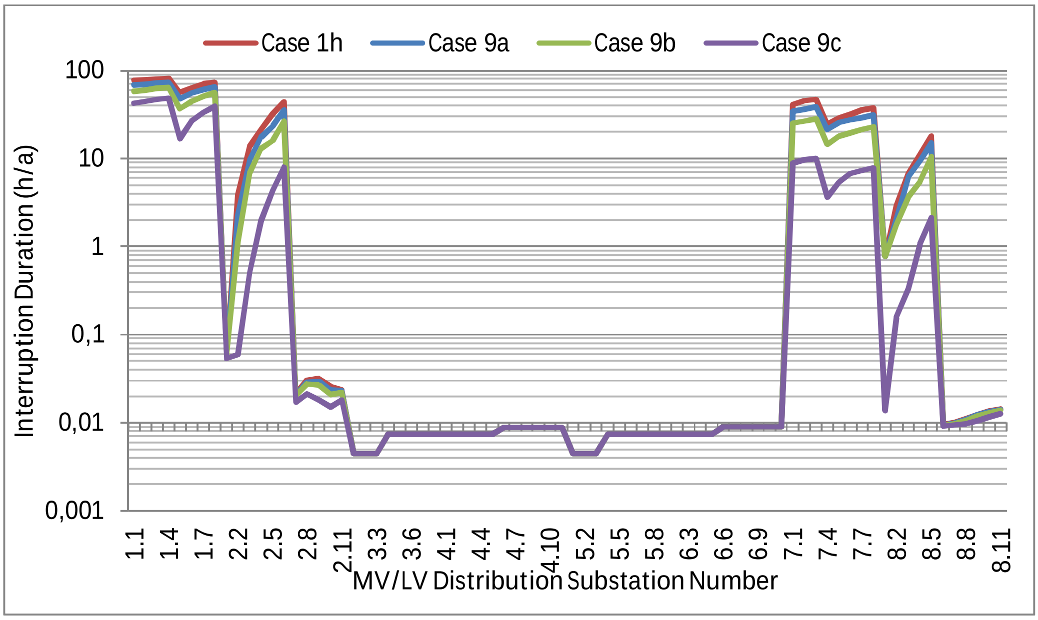

Figure 5-39: Annual interruption duration for MV/LV substations (Case 9a-9c)

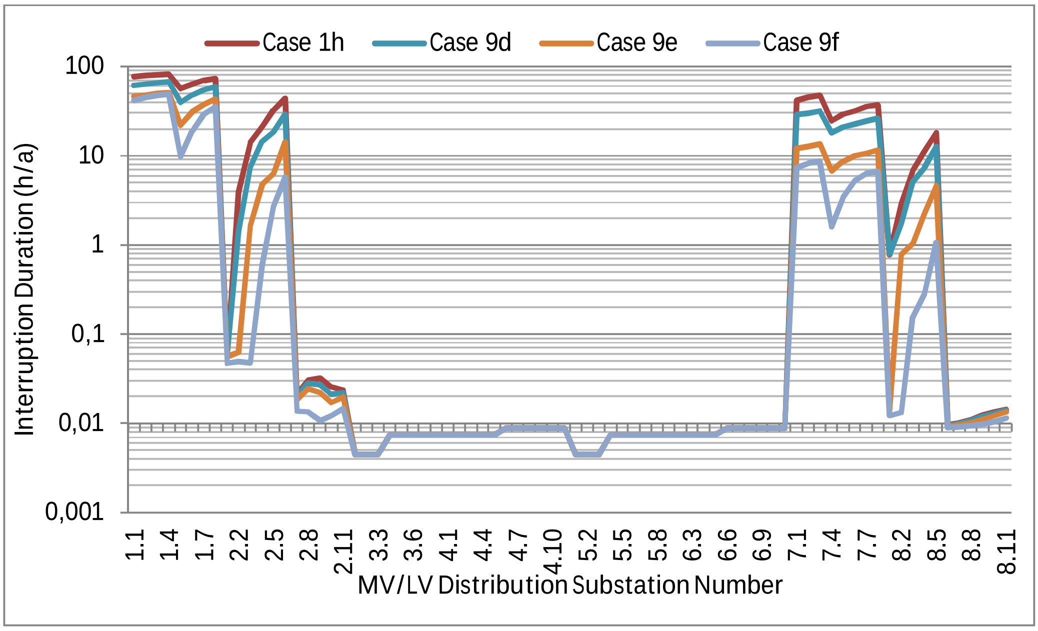

Figure 5-40: Annual interruption duration for MV/LV substations (Case 9d-9f)

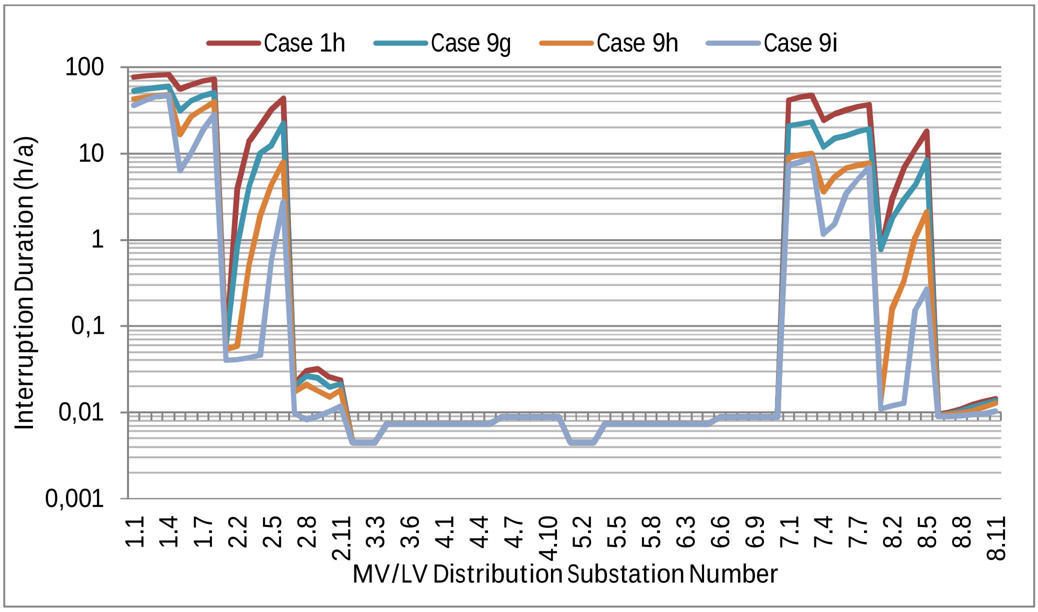

Figure 5-41: Annual interruption duration for MV/LV substations (Case 9g-9i)

5.4.3 Electrical Vehicle (EV) Cases

The effect of Electrical Vehicle (EV) load on power system network is evaluated in this section. For following scenarios decrease in outage cost is calculated.

- EV as DR: Penetration 50% (5% of peak demand due to EV)
- EV as DR: Penetration 100% (10% of peak demand due to EV)

Case 10: EV Penetration 50% as DR

EV load is considered as Demand response. Let for 50% penetration of electrical vehicles, 5% of peak load is due to EVs. Demand postponement time is 21 hours (out of 24h day, 3 h-charging time). Decrease in outage cost is shown in Table 5-13 and Figure 5-42 for various load growth. There is small decrease (maximum 10,93 % for load growth 5%) in outage cost mainly because probability of EV in the network for charging (3/24) is low. Another reason is EVs constitute a small percentage of overall load. Difference in benefit for different load growth is based on ability to reduce requirement of load curtailment or load transfer.

Table 5-13: Decrease in Outage Cost due to EV load (Case 10)

Load Growth (%)	Outage Cost with capacity Increase (€/a)	Outage Cost without Capacity Increase (€/a)	Outage Cost considering EVs (€/a)	Decrease in Outage Cost (%)
5	4 260,36	87 643,05	78 525,42	10,93
10	4 463,24	372 946,10	336 516,50	9,89
15	4 666,11	827 756,15	760 337,51	8,19
20	4 868,99	1 591 934,35	1 461 675,60	8,21
25	5 071,86	2 626 270,67	2 411 499,35	8,19
30	5 274,74	3 766 802,25	3 472 360,61	7,83
40	5 680,49	6 480 238,22	5 981 353,95	7,71
50	6 086,24	9 486 861,76	8 742 095,45	7,86

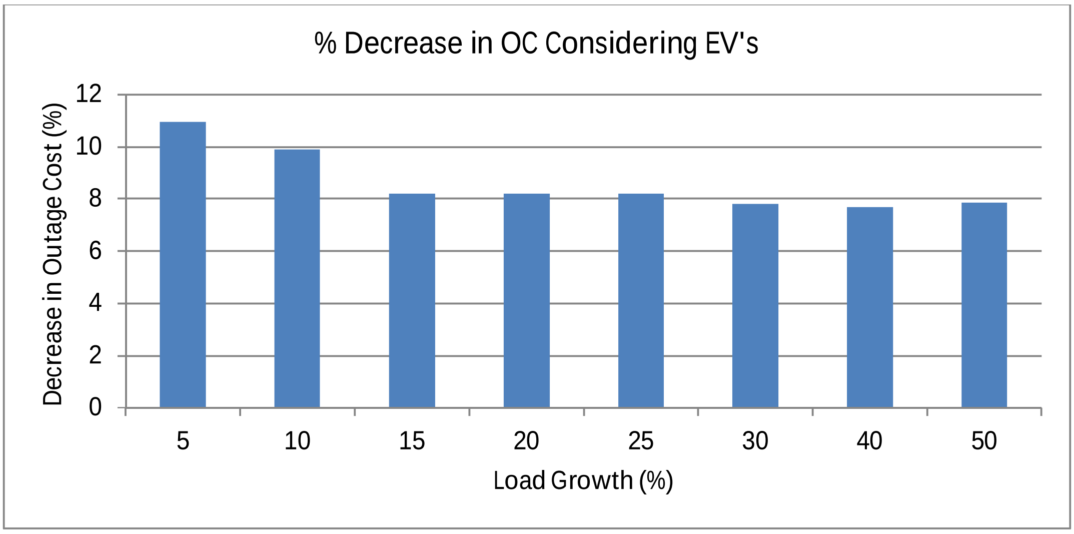

Figure 5-42: Decrease in Outage Cost due to EV load (Case 10)

CASE 11: EV Penetration 100% as DR

Similar to case 10, EV load is considered as Demand response. Let for 100% penetration of electrical vehicles, 10% of peak load is due to EVs. Demand postponement time is 21 hours (out of 24h day, 3 h-charging time). Decrease in outage cost is shown in Table 5-14 and Figure 5-43 for various load growth. There is small decrease (maximum 12,5 % for load growth 5%) in outage cost mainly because probability of EV in the network for charging (3/24) is low. Another reason is EVs constitute a small percentage of overall load. Difference in benefit for different load growth is based on ability to reduce requirement of load curtailment or load transfer. Compared to case 10 only small improvement in decrease in outage cost is observed, which is due to high EV penetration.

Table 5-14: Decrease in Outage Cost due to EV load (Case 11)

Load Growth (%)	Outage Cost with capacity Increase (€/a)	Outage Cost without Capacity Increase (€/a)	Outage Cost considering EVs (€/a)	Decrease in Outage Cost (%)
5	4 260,36	76 782,72	67 720,80	12,50
10	4 463,24	335 120,68	295 210,70	12,07
15	4 666,11	744 875,12	662 312,58	11,15
20	4 868,99	1 431 375,94	1 284 462,96	10,30
25	5 071,86	2 370 927,49	2 137 364,52	9,87
30	5 274,74	3 419 794,30	3 098 782,01	9,40
40	5 680,49	5 916 489,93	5 406 858,50	8,62
50	6 086,24	8 692 092,68	7 977 678,09	8,22

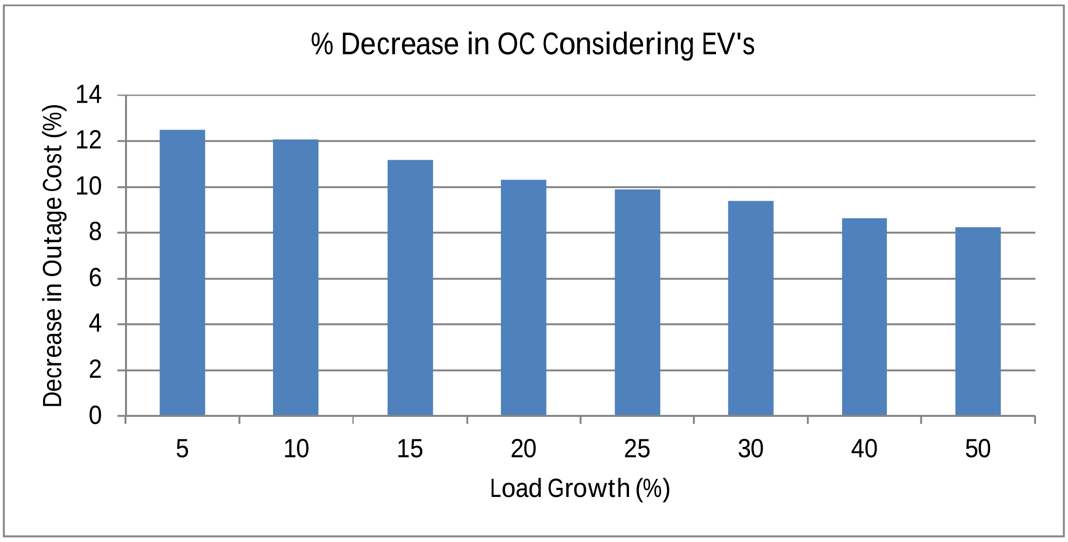

Figure 5-43: Decrease in Outage Cost due to EV load (Case 11)

5.5 DR for Full Compensation of Load Growth

In previous section it was observed that for most of cases considered DR capacity was not able to fully compensate capacity constraint due to load growth. In this section, amount of DR required to fully compensate load growth is calculated.

For fixed DR postponement time and given load growth Eq.4-5 can be modified to find required value.

$$C_{DR} = \frac{24 \cdot Load\ Growth}{T_{DR}} \qquad \textbf{\textit{Eq. 5-1}}$$

Using Eq. 5-1 DR capacity required to fully compensate load growth is shown in Table 5-15. It can be seen that even for low load growth very high DR capacity is required. To compensate 5% load growth: one hour DR postponement time requires 120% DR capacity which is impossible; two hour DR postponement time requires 60% DR capacity which is very high compared to load growth and five hour DR postponement time requires 24% DR capacity. For higher load growth e.g. more than 20%, it is impossible to compensate considering up to five hour DR postponement time.

Table 5-15: DR Capacity required for Full compensation of Load Growth.

Load Growth (%)	DR Postponement Time (h)	Required DR Capacity (%)	Comments
5	1	120	Impossible
5	2	60	
5	5	24	
10	1	240	Impossible
10	2	120	Impossible
10	5	48	
15	1	360	Impossible
15	2	180	Impossible
15	5	72	
20	1	480	Impossible
20	2	240	Impossible
20	5	96	
25	1	600	Impossible
25	2	300	Impossible
25	5	120	Impossible
30	1	720	Impossible
30	2	360	Impossible
30	5	144	Impossible
40	1	960	Impossible
40	2	480	Impossible
40	5	192	Impossible
50	1	1200	Impossible
50	2	600	Impossible
50	5	240	Impossible

6 CONCLUSION AND FUTURE WORK

6.1 Conclusion

The purpose of this thesis is to assess the possibility of reducing reserve requirement of network components in the grid by Demand Response and Electrical Vehicles. The method followed is; to consider load growth, calculate the outage cost without investing into network, calculate the outage cost without investing into network considering DR and EVs, then compare both the outage cost. Different load growth and DR capacities are considered in the analysis.

For a particular load growth benefit of DR depends on DR capacity and postponement time. Even for low load growth (e.g. 5%), to completely mitigate reserve requirement, high capacity DR resources are required. Results are not motivating, however, may be DR along with distributed energy resources (DER) can make reasonable effect. These results can be used in estimating the advantage / disadvantage of delaying investing in network, by comparing cost of adding capacity and outage cost if capacity is not increased with load growth.

It can also be concluded that EVs are not feasible in mitigating the reserve requirements of grid. The main reason is low probability of availability of EVs at the time of requirement. Also Electrical Vehicles constitute only small portion of overall load at grid. However, this does not mean EVs are not useful; these may be helpful in other scenarios e.g. to store energy when excess quantity is available.

6.2 Future Work

The analysis in thesis is conducted for single fault at a time and load point reliability indices are calculated.

- Analysis can be improved by considering effect of multiple contingencies.
- Reliability indices at different system levels shall be calculated.
- Distributed Energy Resources (DER) will be added in further study.
- Load profile of EV’s will be included in future analysis.

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APPENDIX

Markov Process [8]

A stochastic process satisfying the condition of Markov property is called Markov process.

Markov property in words, if present state of process is given, the future state is independent of previous states. Below is the equation form of Markov property

$$
\begin{aligned}
&Pr(X(t+s) = j | X(t) = i, X(u) = x(u), 0 \leq u < s) \\
&= Pr(X(t+s) = j | X(s) = i) \\
&for\ all\ possible\ x(u), 0 \leq u < s
\end{aligned}
\qquad \textbf{\textit{Eq. 1 [8]}}
$$

Where

$\{X(t), t \geq 0\}$ is continuous time stochastic process.

$X = \{0, 1, 2, \ldots, r\}$ is state space.

$X(s) = i$ –State of process at time s is i.

$Pr(X(t+s)) = j$ – Probability that the process will be in state j at time t + s.

$\{x(u), 0 \leq u < s\}$ is history of process.

The transitional probabilities of Markov process may be arranged in matrix form as

$$
P(t) = \begin{bmatrix} P_{00}(t) & P_{01}(t) & \ldots & P_{0r}(t) \\ P_{10}(t) & P_{11}(t) & \cdots & P_{1r}(t) \\ \vdots & \vdots & \ddots & \vdots \\ P_{r0}(t) & P_{r1}(t) & \cdots & P_{rr}(t) \end{bmatrix} \qquad \textbf{\textit{Eq. 2 [8]}}
$$

Where $P_{ij}(t) = Pr(X(t) = j \mid X(0) = i)$ for all $i, j \,\varepsilon\, X$

The entries in row i represents the transitions out of state $(for\ j \neq i)$, and entries in the column j represents the transition into state $j(for\ i \neq j)$. Since all the entities in $P(t)$ are probabilities, so

$$0 \leq P_{ij}(t) \leq 1\ for\ all\ t \geq 0, i, j \varepsilon\ X$$

$$\sum_{j=0}^{r} P_{ij}(t) = 1\ for\ all\ i\ \varepsilon\ X \qquad \textbf{\textit{Eq. 3 [8]}}$$

The amount of time spent in the state i ($\tilde{T}$) before making transition to other state is exponentially distributed (rate$=$ α_i). The mean sojourn time in state i is

$$E(\tilde{T}) = \frac{1}{\alpha_i}$$

The transition rate from state i to j is defined as

$$a_{ij} = \alpha_i * P_{ij} \quad for\ all\ i \neq j \qquad \textbf{\textit{Eq. 4 [8]}}$$

Since $\sum_{j\neq i}^{r} P_{ij} = 1$, therefore

$$\alpha_i = \sum_{j=0, j\neq i}^{r} a_{ij} \qquad \textbf{\textit{Eq. 5 [8]}}$$

For a given a_{ij} other two quantities $\alpha_i$and P_{ij} can be found, thus a Markov process can be defined by state space and transition rates. Transition rates arranged in matrix form A is called transition rate matrix of Markov process.

$$A = \begin{bmatrix} a_{00} & a_{01} & \cdots & a_{0r} \\ a_{10} & a_{11} & \cdots & a_{1r} \\ \vdots & \vdots & \ddots & \vdots \\ a_{r0} & a_{r1} & \cdots & a_{rr} \end{bmatrix} \qquad \textbf{\textit{Eq. 6 [8]}}$$

For diagonal elements special notion used is given below

$$a_{ii} = -\alpha_i = -\sum_{j=0, j\neq i}^{r} a_{ij} \qquad \textbf{\textit{Eq. 7 [8]}}$$

Let T_{ij} is the time process spends in state i before transition to state $j (j \neq i)$. The time is exponentially distributed with rate a_{ij}. Considering small time interval Δt we have

$$P_{ii}(\Delta t) = \Pr(\breve{T_\iota} > \Delta t) = e^{-\alpha i \Delta t} \approx 1 - \alpha_i \Delta t$$
$$P_{ij}(\Delta t) = \Pr(T_{ij} \leq \Delta t) = 1 - e^{-aij\Delta t} \approx a_{ij} \Delta t$$

From these equations we find $a_{ij}$and α_i

$$\lim_{\Delta t \to 0} \frac{1 - P_{ii}(\Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{Pr(\breve{T\iota} < \Delta t)}{\Delta t} = \alpha_i \qquad \textbf{\textit{Eq. 8 [8]}}$$

$$\lim_{\Delta t \to 0} \frac{P_{ij}(\Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{Pr(T_{ij} < \Delta t)}{\Delta t} = a_{ij} \quad for\ i \neq j \qquad \textbf{\textit{Eq. 9 [8]}}$$

By using Markov property and the law of total probability Chapman-Kolmogorov equations are

$$P_{ij}(t + \Delta t) = \sum_{k=0}^{r} P_{ik}(\Delta t) P_{kj}(t)$$

Splitting interval $(0, t + \Delta t)$ in two parts: transition from state i to k in small interval $(0, \Delta t)$and transition from state k to j in remaining interval. Now consider

$$P_{ij}(t + \Delta t) - P_{ij}(t) = \sum_{k=0, k\neq i}^{r} P_{ik}(\Delta t) P_{kj}(t) - [1 - P_{ii}(\Delta t)]\, P_{ij}(t)$$

Dividing by Δt and taking limit as $\Delta t \to 0$

$$\lim_{\Delta t \to 0} \frac{P_{ij}(t + \Delta t) - P_{ij}(t)}{\Delta t} = \lim_{\Delta t \to 0} \sum_{k=0, k \neq i}^{r} \frac{P_{ik}(\Delta t)}{\Delta t} P_{kj}(t) - \alpha_i P_{ij}(t) \qquad \textbf{\textit{Eq. 10 [8]}}$$

Since the summing index is finite, interchanging the limit and summation on the RHS and using Eq.8 & Eq.9 along with $\dot{P}_{ij}(t) = \frac{d}{dt} P_{ij}(t)$, we get Kolmogorov backward equations.

$$\dot{P}_{ij}(t) = \sum_{k=0, k \neq i}^{r} a_{ik} P_{kj}(t) - \alpha_i P_{ij}(t) = \sum_{k=0}^{r} a_{ik} P_{kj}(t) \qquad \textbf{\textit{Eq. 11 [8]}}$$

In matrix form

$$\dot{\boldsymbol{P}}(t) = A \cdot \boldsymbol{P}(t) \qquad \textbf{\textit{Eq. 12 [8]}}$$

Similarly Kolmogorov forward equations are

$$\dot{\boldsymbol{P}}(t) = \boldsymbol{P}(t) \cdot A \qquad \textbf{\textit{Eq. 13 [8]}}$$

Assuming initial state of process is $i(t = 0) i.e. X(0) = i$

So $P_i(0) = 1$ and $P_k(0) = 0$

As initial state is known, notion $P_{ij}(t)$ may be simplified as $P_j(t)$. The vector $\boldsymbol{P}(t) = [\mathrm{P}_0(\mathrm{t}), \mathrm{P}_1(\mathrm{t}), \ldots, \mathrm{P}_\mathrm{r}(\mathrm{t})]$ denotes the distribution of process at time t. The new form of equations from Kolmogorov forward equations will be

$$[P_0(t) \quad P_1(t) \quad \cdots \quad P_r(t)] \cdot \begin{bmatrix} a_{00} & a_{01} & \cdots & a_{0r} \\ a_{10} & a_{11} & \cdots & a_{1r} \\ \vdots & \vdots & \ddots & \vdots \\ a_{r0} & a_{r1} & \cdots & a_{rr} \end{bmatrix} = [\dot{P_0}(t) \quad \dot{P_1}(t) \quad \cdots \quad \dot{P_r}(t)] \qquad \textbf{\textit{Eq. 14 [8]}}$$

Another form

$$\dot{\boldsymbol{P}}(t) = \boldsymbol{P}(t) \cdot A \qquad \textbf{\textit{Eq. 15 [8]}}$$

Since the sum of entries in each row in A is zero, matrix is singular. Consequently above equations does not have unique solution. By using fact $\sum_{j=0}^{r} P_j\,(t) = 1$ and initial state, often solution can be found.

Considering steady state probabilities i.e.$(\mathrm{t} \to \infty)$: $\lim_{t\to\infty} P_j(t) = P_j$ tends to constant thus derivative is zero. Hence

$$0 = \boldsymbol{P} \cdot A \qquad \textbf{\textit{Eq. 16 [8]}}$$

With $\sum_{j=0}^{r} P_j = 1$ and r out of $r + 1$ equations from above Eq. 16 steady state probabilities can be calculated.

Visit Frequency:

By putting values for steady state situation Kolmogorov forward equations can be written as

$$0 = \sum_{k=0, k\neq j}^{r} a_{kj}\, P_k - \alpha_j\, P_j$$

$$P_j \alpha_j = \sum_{k=0, k\neq j}^{r} P_k a_{kj} \qquad \textbf{\textit{Eq. 17 [8]}}$$

The probability of departure from state j in the time interval $(t, t + \Delta t)$ is

$$\sum_{k=0,k\neq j}^{r} Pr\big((X(t+\Delta t)=k)\cap(X(t)=j)\big)$$
$$= \sum_{k=0,k\neq j}^{r} Pr\big((X(t+\Delta t)=k)|(X(t)=j)\big)\cdot Pr(X(t)=j) \qquad \textbf{\textit{Eq. 18 [8]}}$$
$$= \sum_{k=0,k\neq j}^{r} P_{jk}(\Delta t)\cdot P_j(t)$$

For steady state $(t \to \infty)$ frequency of departure from state j is given as

$$v_j^{dep} = \lim_{\Delta t\to 0}\frac{\sum_{k=0,k\neq j}^{r} P_{jk}(\Delta t)\cdot P_j}{\Delta t} = P_j\alpha_j \qquad \textbf{\textit{Eq. 19 [8]}}$$

Similarly, the frequency of departure from state k into j is $P_k a_{kj}$. Therefore, total frequency of arrival into state j is

$$v_j^{arr} = \sum_{k=0,k\neq j}^{r} P_k\, a_{kj} \qquad \textbf{\textit{Eq. 20 [8]}}$$

Above two equations show that frequency of departure from state j is equal to frequency of arrival into state j. Thus steady state visit frequency to state j is defined as

$$v_j = P_j\alpha_j = \sum_{k=0,k\neq j}^{r} P_k\, a_{kj} \qquad \textbf{\textit{Eq. 21 [8]}}$$

Mean Duration Of Visit:

It is given as reciprocal of rate of exponentially distributed function.

$$\Theta_j = \frac{1}{\alpha_j} \quad for \ j = 0,1, \dots, r$$ ***Eq. 22 [8]***

By putting value of $\alpha_j$from visit frequency Eq. 21

$$\Theta_j = \frac{P_j}{v_j} \quad for \ j = 0,1, \dots, r$$ ***Eq. 23 [8]***